PLATE 1

DRAWING EQUIPMENT

PRACTICAL GEOMETRY
FOR
JUNIOR BUILDERS

BY

LESLIE A. LEE, L.I.O.B., M.R.San.I., A.I.S.E., A.B.I.C.C.
City and Guilds Hons.—Silver Medallist (Carpentry and Joinery)
SENIOR ASSISTANT THE POLYTECHNIC SCHOOL OF ARCHITECTURE, SURVEYING AND BUILDING, LONDON

AND

R. FRASER REEKIE, F.R.I.B.A., Dip. Arch., Architect
R.I.B.A. Bossom Gold and Silver Medallist
R.I.B.A. Grissell Gold Medallist (Advanced Construction)
FORMERLY INSTRUCTOR IN DRAUGHTSMANSHIP, LECTURER IN DESIGN AND CONSTRUCTION THE POLYTECHNIC SCHOOL OF ARCHITECTURE, SURVEYING AND BUILDING, LONDON

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PREFACE

The contents of this book have been carefully considered to give beginners in the Building Trades, the Architectural and Surveying professions and similar bodies, the thorough grounding in the principles of Plane and Solid Geometry necessary in the Drawing Office and in practical applications.

A knowledge of Plane and Solid Geometry is invaluable to those engaged in Building and its allied branches, and good workmanship will not result from the rule-of-thumb methods sometimes, alas, adopted by the technician or tradesman.

The authors' wide practical and technical experience has led them to apply the majority of the examples in this book to some definite case. The diagrams and text have been kept as clear and as simple as possible, and it is to be hoped that by making a complete study of the contents, readers will be able to follow the progressive stages of Geometry as applied to practice. (This can be taken still further by reference to "Descriptive Geometry for Architects and Builders"—Lee and Reekie—also published by Messrs. Edward Arnold.)

Every effort has been made to cover the syllabus of the Junior Technical School of Building, also that of the 1st year of the Senior National Diploma and Certificate Courses. There are also to be found numerous examination questions relating to the various chapters.

We wish to offer our thanks for the valuable help given in the preparation of drawings to Miss P. K. Thorne, and to our colleague, A. Turner, B.Sc., for his criticism.

Leslie A. Lee
R. Fraser Reekie

CONTENTS

PLATES

CHAPTER I

DRAWING EQUIPMENT

THE drawings necessary for carrying out the exercises in this book should be made with great care and accuracy. Good drawings cannot be made without satisfactory equipment. Plate I shows the main items, which are further described below.

Drawing-board. This should have a flat even surface, and perfectly true edges. (Laminated boards, which are unlikely to shrink or warp, are recommended.) For the exercises here described, half-imperial size (22″×15″) will be large enough, although full imperial size is more convenient (22″×30″).

Tee-square, preferably of mahogany, with ebony or celluloid ruling edge, which should be bevelled. Choose a size to fit the drawing-board, i.e. half-imperial or imperial.

Set-squares. Either two plain celluloid ones of 45 degrees and 60 degrees respectively, or an adjustable set-square which can be set to any angle should be chosen. Set-squares should have sides of not less than 6″ length, and should be square-edged, *not bevelled.*

Protractor. This is not essential, but is sometimes useful for measuring or setting-out angles, if an adjustable set-square is not available.

Drawing Instruments. These are probably the most costly items of equipment as good instruments are essential. They can be bought in sets, or purchased separately. The latter is generally the better method, and less expensive. Required are: dividers, 4″ or more in length; compasses with interchangeable pencil, pen and divider points; spring-bow dividers; spring-bow pen and pencil compasses; and ruling pen for ink lines.

Scale. 12″ or 6″ long, made of boxwood or ivory, and divided into units and twelfths of $\frac{1}{8}''$, $\frac{1}{4}''$, $\frac{3}{8}''$, $\frac{1}{2}''$, $\frac{3}{4}''$, 1″, $1\frac{1}{2}''$ and 3″.

Pencils. Best quality only should be used. If properly sharpened and handled, grades HB and H will meet all normal purposes. Points should be about $\frac{3}{4}''$ long, of which $\frac{1}{4}''$ should be exposed lead, sharpened to a round point. A penknife is best for sharpening. The point will last longer if the pencil is revolved slowly when ruling lines.

Erasers. Soft rubbers should be used for correcting pencil errors; special hard rubbers are used for ink lines.

Drawing Papers. There are many kinds and qualities of drawing papers. Good-quality cartridge paper is satisfactory for the work described in this book, and for most uncoloured pencil and ink drawings.

Measuring Equipment

For measuring distances, areas and solids, the following may be used (see Plate II):

(*a*) *Rules* of various kinds, e.g. divided into inches and parts of an inch, or divided into feet and inches. (This includes 2′ 0″, 3′ 0″ and 5′ 0″ folding rules.)

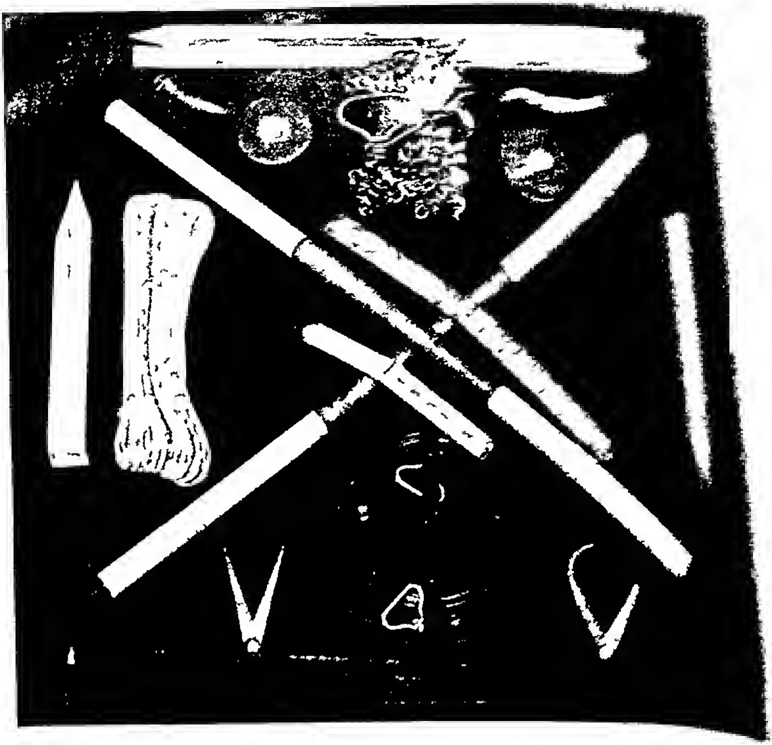
(*b*) *Tapes*, measuring feet and inches, of either steel or linen. (Steel type is more reliable.)

(c) "*Chain*," either of 66′ 0″ in links (Gunter's chain), or 50′ 0″ or 100′ 0″ in feet (Engineers' chain).

(*d*) *Line or Cord*, used for setting out the lines of trenches, walls, etc.

(*e*) *Callipers*, used for measuring circular objects.

In addition, (*f*) *Steel Arrows*, in sets of 10, (*g*) *Pegs*, (*h*) *Stakes*, (*i*) *Survey Poles* are used in land surveying, and the setting out of building sites.



MEASURING EQUIPMENT

CHAPTER II

DEFINITIONS

Terms used in Geometry and Geometrical Drawing

1. **Point.** Although having position, a point has no magnitude, and is merely a small visible mark or dot, geometrically occupying no space. The ends or extremities of lines are points, as is also the intersection of any number of lines.
2. **Line.** A line is considered to have length but no breadth. A drawn line, which must have breadth, is mathematically incorrect, so the middle of such a line is taken in practice as the real line. Lines may be described as "straight" "curved", "mixed", "concave" and "convex".
3. **Straight Line.** A straight line is the shortest distance ruled between two points. (*a*) A straight line is perpendicular to a plane when it is perpendicular to all straight lines which meet it in that plane (Fig. 1, line *AB*). (*b*) A straight line is parallel to a plane when the two do not meet each other even when produced to infinity.
4. **Curved Line.** A curved line is one which does not lie in a straight direction between its extremities, but is continually changing in direction. It can be regularly or irregularly curved, as illustrated in Figs. 2*a* and 2*b*.
5. **Mixed or Compound Line.** A mixed line is one formed by any continuous combination of straight and curved lines, e.g. the outline of a moulding, as in Fig. 3.

Fig. 1

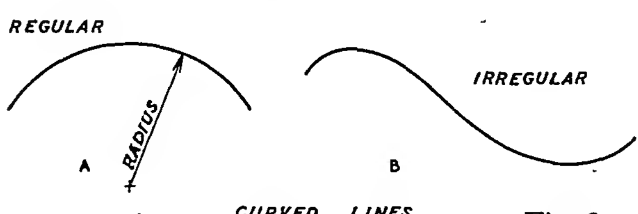


Fig. 2

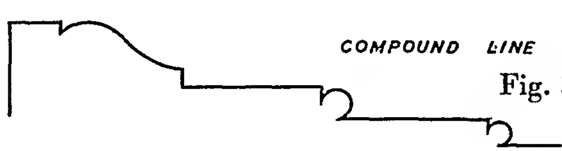


Fig. 3

6. **Concave or Convex Lines.** A concave or convex line is one that cannot be cut by a straight line in more than two points, and which is part of the outline of an area. The difference between concave and convex is illustrated in Fig. 4.
7. **Parallel Lines.** Parallel lines are lines which are an equal distance apart throughout their lengths, and however far extended never meet. They can be straight or curved, as in Fig. 5.
8. **Converging Lines.** Converging lines are straight lines which, if extended in the same plane, can be made to meet or cross one another (see Fig 6).
9. **Surface.** A surface has length and breadth, but no thickness, e.g. the length and breadth of a shadow can be measured, but it has no thickness or substance. The amount of space within the outline of a surface is termed the *area.*
10. **Plane Surface.** A plane surface is a flat surface which coincides with a straight line connecting any two points within it.
11. **Plane Figure.** A plane figure is a representation in outline of a plane surface, and may be bounded by straight or curved lines. If bounded by straight lines it is termed a rectilinear figure, if by curved lines, curvelineal figure.
12. **Parallel Planes.** These are such as do not meet each other, even when produced.
13. **Orthographic Projection.** (*a*) The orthographic projection of a point on to a plane is the base of the perpendicular from the point to the plane. The perpendicular is called the *projector,* and the plane is called the *plane of projection.* (*b*) The orthographic projection of a given line on to a plane is the line generated by the base of a perpendicular to the

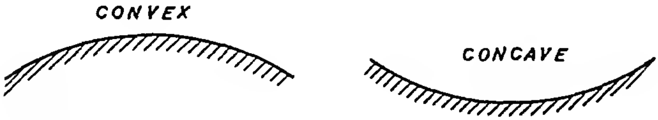
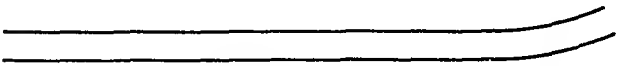
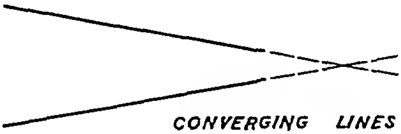


Fig. 4



PARALLEL LINES

Fig. 5



CONVERGING LINES

Fig. 6

plane, which perpendicular moves so as always to intersect the given line. The surface generated by the moving perpendicular is called the *projecting surface.* When the line which is projected is straight, the projecting surface is called the *projecting plane* (see Chapter XIV).

14. **Angles.** Angles are formed by two straight lines converging to a point, or they may be defined as the space swept out by a line rotating about one extremity. Fig. 7 gives an angle of 23 degrees when the line has rotated from BC to BA.

15. **Right Angle.** A right angle is one where the included angle between two straight lines is 90 degrees. Fig. 8 shows a line AB at right angles to CD, or AB is said to be perpendicular to CD at B. In Fig. 9 the angles are right angles.

16. **Obtuse Angle.** An obtuse angle is one where the included angle between two straight lines is greater than 90 degrees (Fig. 10).

17. **Acute Angle.** An acute angle is one where the included angle between two straight lines is less than 90 degrees (Fig. 11).

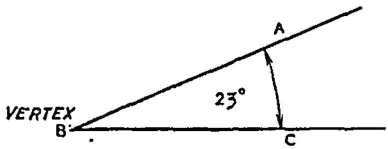


Fig. 7

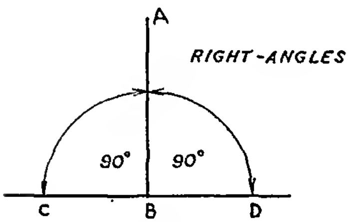


Fig. 8

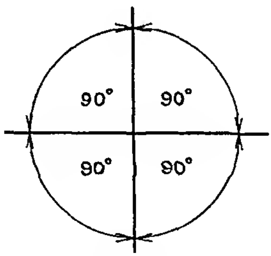


Fig. 9

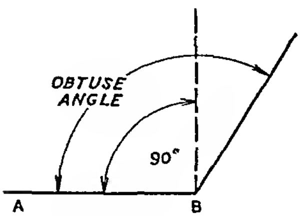


Fig. 10

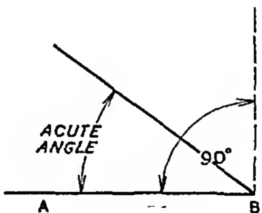


Fig. 11

NOTE.—The number of degrees by which an acute angle is less than 90 degrees is termed the *complement* of the angle, and the difference between an obtuse angle and 180 degrees is termed the *supplement* of that angle.

18. **Dihedral Angle.** The dihedral angle is the angle between two planes, and is measured by the angle formed by two straight lines, drawn from a point in their intersection, each perpendicular to the intersection, and lying one in each plane (see also Chapter XVI).
19. **Triangle.** A triangle is a plane figure bounded by three straight lines, or three points or objects not in one straight line, together with the imaginary lines joining them (see also Chapter IV).
20. **Right-angled Triangle.** A right-angled triangle is one in which one of the angles is a right angle. The *hypotenuse* is the side opposite the right angle and is the longest side of the triangle (Fig. 12).
21. **Quadrilateral Figures.** These are plane figures having four straight sides. They are also termed *quadrangles* because they contain four angles.
22. **Parallelogram.** A parallelogram is a four-sided rectilineal figure whose opposite sides are parallel to each other (see also Chapter V).
23. **Rectangle.** A rectangle, sometimes called an *oblong*, is a parallelogram having four right angles (Fig. 13).
24. **Square.** A square is a rectangle with all four sides equal and the four angles are right angles (Fig. 14).
25. **Rhomboid.** A rhomboid is a parallelogram in which the adjoining sides are unequal, and the angles are not right angles, but the opposite angles are equal (Fig. 15).
26. **Rhombus.** A rhombus is a parallelogram in which all four sides are equal, but the angles are not right angles (Fig. 16).
27. **Trapezoid.** A trapezoid is a plane figure with two opposite sides parallel, but the remaining two sides are neither parallel nor equal (see Fig. 17).
28. **Trapezium.** This term is often used where *trapezoid* is meant. It is an irregular quadrilateral having no two sides parallel (Fig. 18).
29. **Diagonal.** A straight line drawn between opposite angular points—i.e. running across a surface obliquely to its sides (see Fig. 13).
30. **Polygon.** A polygon is any plane figure bounded by more than four straight lines. (*a*) A *regular polygon* has all its sides and interior angles equal, (*b*) an *irregular polygon* has unequal interior angles, the sides may be equal or unequal (see Chapter VI).

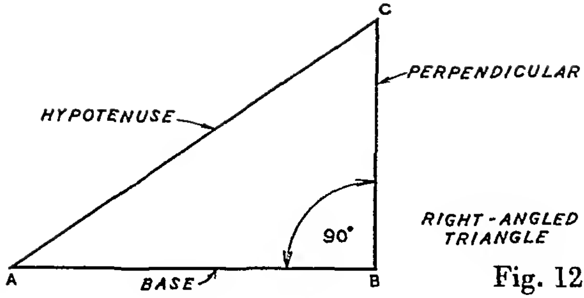


Fig. 12

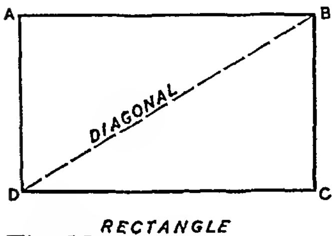


Fig. 13

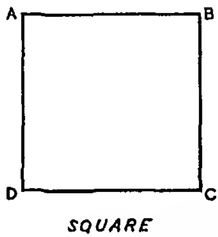


Fig. 14

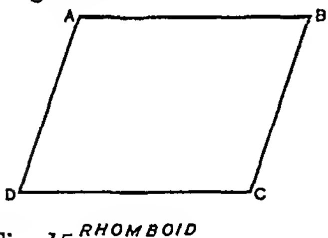


Fig. 15

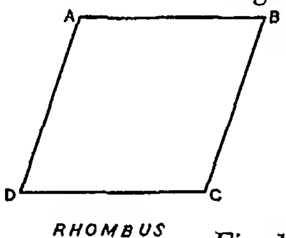


Fig. 16

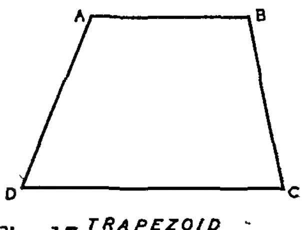


Fig. 17

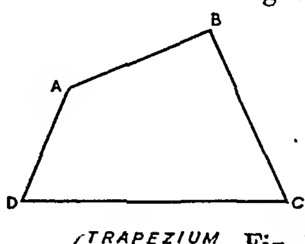


Fig. 18

31. **Circle.** A circle is a plane figure bounded by one uniformly curved line, known as the circumference, which is equidistant throughout its length from the centre (see Chapter VII).
32. **Concentric Circles.** These are circles struck from the same centre, having parallel circumferences (Fig. 19).
33. **Eccentric Circles.** These are circles having different centres, but one circle lying within the other (Fig. 20).
34. **Altitude.** The vertical height of any figure is called its altitude. To find the altitude, draw a line from the highest point of the figure, perpendicular to the base line, or the base line produced. In Fig. 21 the line AB is the altitude of the triangle.
35. **Solid.** A solid is any body having three dimensions, length, breadth and thickness. The term can be applied to anything that has substance.
36. **Polyhedral Angle** is formed when three or more planes meet in a point. It consists of as many plane angles and of as many dihedral angles as there are planes.
37. **Polyhedron.** A polyhedron is a solid bounded by plane surfaces called *faces* which meet in straight lines called *edges.*
38. **Prism.** A prism is a polyhedron with side faces which are parallelograms, and end faces which are equal triangles, quadrilaterals or polygons, in parallel planes. A straight line passing through the centres of the end faces is known as the *axis.* If the axis is at right angles to the end faces the solid is known as a *right prism*; if it is not at right angles, the solid is called an *oblique prism* (see Fig. 22).
39. **Pyramid.** A pyramid is a polyhedron having one face, the *base*, which is a triangle, quadrilateral or polygon, and other faces which are triangles all having the same vertex. The axis of the pyramid is a straight line passing through the vertex and the centre of the base (Fig. 23, line AB). A pyramid having a triangular base is known as a *tetrahedron.* A *regular tetrahedron* is a solid enclosed by four equal and equilateral triangular faces.
40. **Cube.** A cube is a solid with six equal squares for its faces.
41. **Octahedron.** A regular octahedron has eight equal and equilateral triangles for its faces (see Chapter XIX).
42. **Dodecahedron.** A regular dodecahedron has twelve equal and regular pentagonal faces (see Chapter XIX).
43. **Icosahedron.** A regular icosahedron is a solid having twenty equal and equilateral triangles for its faces, all its dihedral angles being equal.
44. **Surface of Revolution.** The surface generated by the rotation of a straight or curved line or *generator* about a fixed straight line or axis, is known as a surface of revolution.

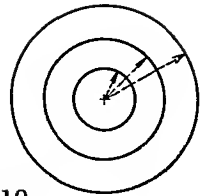


Fig. 19
CONCENTRIC CIRCLES

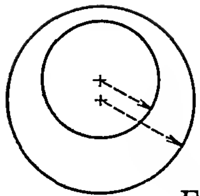
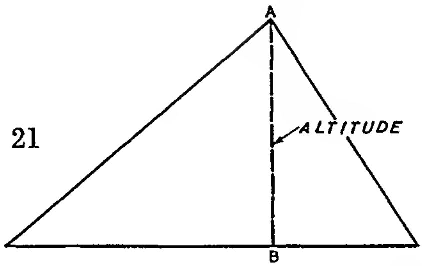


Fig. 20
ECCENTRIC CIRCLES

Fig. 21



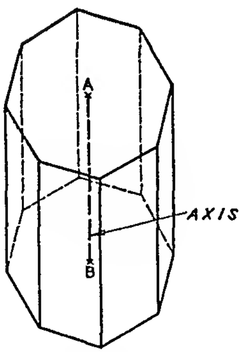


Fig. 22

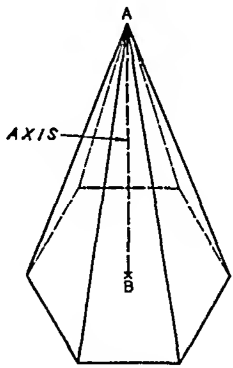


Fig. 23

45. **Right Cone.** A right eone is a solid, the surfaees of whieh are formed by the rotation of a right-angled triangle about the base or perpendieular as axis (Fig. 24).

46. **Cylinder.** A eylinder is a solid the surfaees of whieh are generated by the rotation of a reetangle about one of its sides as axis.

47. **Sphere.** A sphere is a solid, the curved surfaee of which is generated by the rotation of a semieirele about its diameter as axis. The centre of the sphere is the point whieh is equidistant from all points on the surfaee.

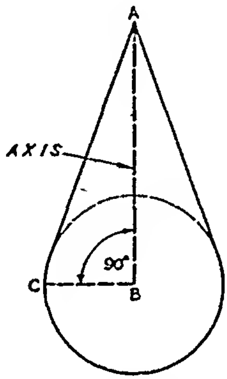


Fig. 24

CHAPTER III

SCALES

DIVISION OF LINES; CONSTRUCTION OF LINEAR, DIAGONAL AND VERNIER SCALES; SCALE OF CHORDS

IN practical Geometry and in drawing plans, elevations and sections of objects and buildings it is necessary to use scales. Scales are constructed by dividing straight lines into appropriate units of length proportionate to actual units of measurement.

Division of Lines

Fig. 25 shows how any line *AB* can be divided into a number of equal parts. The method is to draw a line at an acute angle from *A*, and along this to plot equal units of any convenient dimension—8 are shown in the example—by the use of dividers or scale. From the end of the last of these units, marked *C*, a line is drawn to *B*, and parallel to it other lines are drawn from the intermediate points dividing *AB* into 8 equal parts.

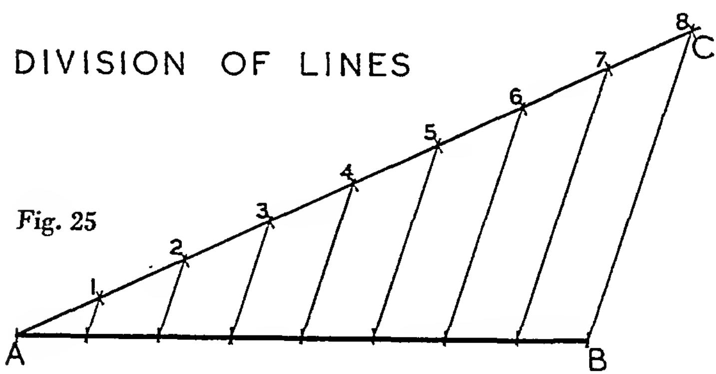


Fig. 25

Fig. 26 shows another method. Let the line AB be the given length of line which is to be divided into, say, five equal parts. Draw the straight line CD by stepping off five equal units of any dimension, indexed 1 to 5. With centres C and D and radius $C5$, describe arcs to find E. Draw lines from C, 1, 2, 3, 4, and 5 to meet at E. Mark point F along EC so that EF equal AB and draw FG parallel to CD. FG is also equal to line AB, and is divided into 5 equal parts.

Fig. 27 shows how a length of board of any width can be divided into a number of equal units by placing a rule diagonally across it so that convenient equal divisions can be marked off.

Fig. 28 shows how any straight line can be bisected (divided into two equal parts). Using compasses and taking A and B, the ends of the line, as centres, and with any radius greater than half AB, two arcs are drawn to intersect on either side of the line A straight line joining these intersections bisects AB.

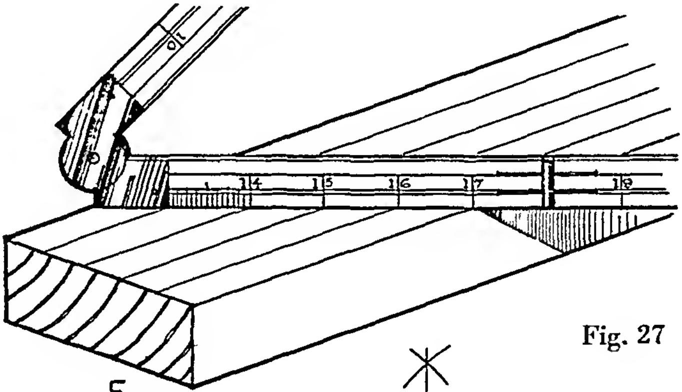


Fig. 27

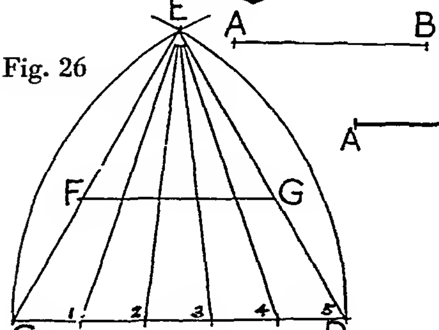


Fig. 26

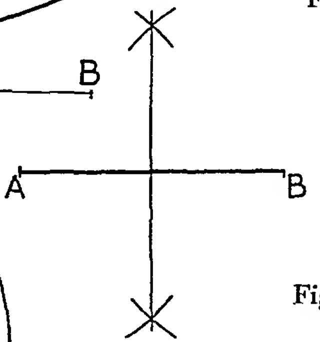


Fig. 28

Construction of Scales

Fig. 29 shows examples of common scales in which inches or parts of an inch represent larger units of measurement:

$\frac{1}{8}''$	equals 1′ 0″ (8 ft. to 1 in.)	$\frac{1}{96}$	representative fraction
$\frac{1}{2}''$	equals 1′ 0″ (2 ft. to 1 in.)	$\frac{1}{24}$	,, ,,
1″	equals 1′ 0″ (1 ft. to 1 in.)	$\frac{1}{12}$	,, ,,
$1\frac{1}{2}''$	equals 1′ 0″	$\frac{1}{8}$	,, ,,
3″	equals 1′ 0″	$\frac{1}{4}$	,, ,,
1″	equals 1 mile		
1″	equals 10 paces		

In the construction of these scales it may be found simplest to rule straight lines and mark off the required units from a rule or draughtsman's scale. As shown, the first unit of each scale is subdivided into a number of fractions; for example, in the scale of $1\frac{1}{2}''$ equals 1′ 0″, the first unit is divided into twelfths—each actually $\frac{1}{8}''$—representing inches.

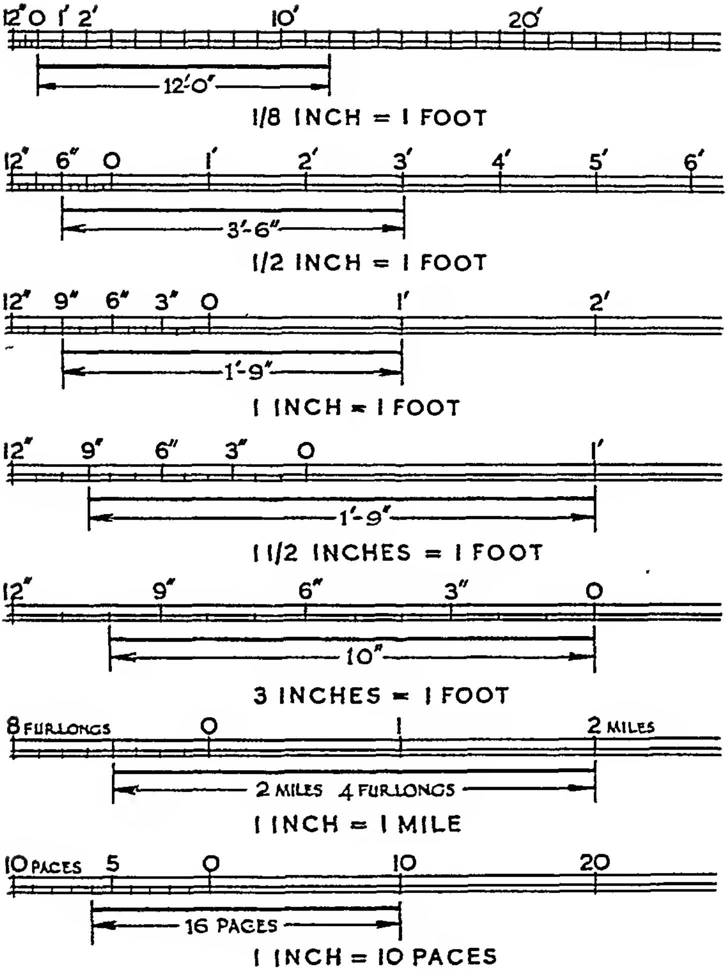
(Note: A definite reading has been indicated on each scale.)

The following are the usual scales for architects', builders' and surveyors' drawings:

For lay-outs, site plans, 41·6 ft. to 1 in. ($\frac{1}{500}$ natural scale).	
For plans, elevations and sections of large buildings,	8 ft. to 1 in.
For ,, ,, ,, small ,,	4 ft. to 1 in.
For details, particularly working drawings	2 ft. to 1 in.
For isolated details	1 ft. to 1 in.
For workshop drawings, Full Size Details (F.S.D.) are required.	

As used on Ordnance Maps

General Map of the United Kingdom,	1 in. to 1 mile ($\frac{1}{63360}$ natural scale)
County Maps,	6 ins. to 1 mile ($\frac{1}{10560}$,, ,,)
Cadastral or Parish Maps,	25·344 ins. to 1 mile ($\frac{1}{2500}$,, ,,)
	60 ins. to 1 mile ($\frac{1}{1056}$,, ,,)
	120 ins. to 1 mile ($\frac{1}{528}$,, ,,)
	126·720 ins. to 1 mile ($\frac{1}{500}$,, ,,)



SCALES Fig. 29

The Vernier Scale

By means of this type of scale small fractions of a unit can be read. It is adapted for various purposes. For example, the theodolite, a surveying instrument used for measuring angles, has commonly a dial marked with units of degrees and minutes, and a vernier attachment divided into units of 20 seconds, making it possible to measure angles in degrees, minutes and 20 seconds. A circle is divided into 360 parts on the circumference, each division being called a degree. A degree may be divided into 60 parts, each called a minute. A minute may be divided into 60 parts called seconds. With certain types of theodolites it is possible to obtain readings to the fraction of 10 seconds. Weight-testing machines similarly have vernier scales usually to read tons, hundredweights and pounds.

Fig. 35 illustrates the working of the vernier scale. The main scale is a simply constructed scale divided into inches and tenths of an inch. The vernier is set out accurately on a separate part and made 9/10″ long, and is divided into 10 equal parts. It is made to slide under the main scale by being inserted in the slits indicated. If 0 on the vernier is under 0 on the main scale, then 10 on the vernier will coincide with 9 on the main scale. If the vernier is moved along until, say, the seventh unit coincides exactly with some division on the main scale, the reading (the distance from the end of the vernier to the previous subdivision on the main scale) will be 7/10 of 1/10, i.e. 7/100″ or ·07 inches. The reading is always found by noting which division on the vernier exactly coincides with a division on the main scale.

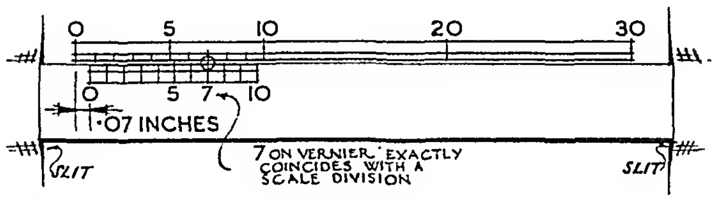


Fig. 35

A useful exercise is the construction of a vernier scale to read degrees, minutes and 20 seconds, as used on the theodolite. Firstly, a suitable radius is determined and the arc of a circle described. On it are marked, using a protractor for quickness, units of degrees, which are subdivided into 3 equal units of 20 minutes. The vernier is made with its length 20 minutes less than 20 degree units. It is divided into 20 equal units, which are again each subdivided into 3 equal units representing 20 second divisions. Fig. 36A shows such a scale with the vernier placed to read 20 seconds. Fig. 36B shows the vernier to read 4 degrees 14 minutes 0 seconds (the reading can be seen to lie between 4 degrees and 4 degrees 20 minutes; division 14 on the vernier exactly coincides with a division on the main scale; therefore 142×0 seconds or 14 minutes 40 seconds is added to the 4 degrees.)

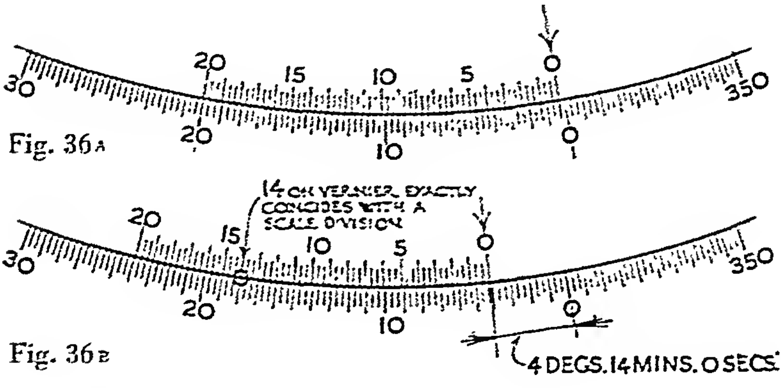


Fig. 36A

Fig. 36B

VERNIER SCALES

Scale of Chords

This scale can be used for setting out any required angle or for measuring the number of degrees of any angle, but is seldom used owing to its inaccuracy.

Fig. 37 shows a scale of chords. The scale is constructed by drawing the quadrant of a circle (quarter circle) with any convenient radius, hence AB equals AC. The arc CB is divided into 9 equal units by using a protractor, or first into 3 equal units using 60°–30° set-square and then into smaller units by trial and error, using dividers. Each unit represents 10 degrees; further subdivision into single degrees can be made by bisection or by using dividers. Using compasses, with B as centre, arcs of circles are drawn from the divisions to the line AB, and thence vertically to the scale.

To illustrate the use of the scale, if two lines AB, AC are set out to form any acute angle, as shown in Fig. 38, and by means of compasses or dividers the length of the scale 0 degrees to 60 degrees is marked along them from A, then the distance from B to C measured and read on the scale gives the angle formed by the lines.

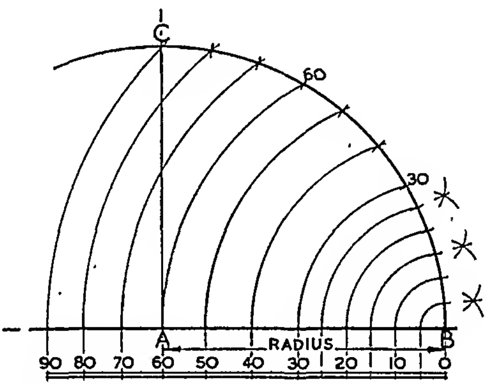


Fig. 37

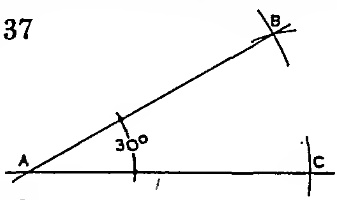


Fig. 38

SCALE OF CHORDS

CHAPTER IV

LINES, ANGLES AND TRIANGLES

SETTING UP AND MEASURING; APPLICATION TO LAND SURVEYING; VARIOUS TYPES OF TRIANGLES; PRISMATIC COMPASS; BOX SEXTANT; THEODOLITE; PLANE TABLE; APPLICATIONS

LINES are normally drawn with the aid of the tee-square if horizontal or along the edge of a set-square if vertical or inclined. Angles are set out using a protractor or adjustable set-square.

Examples of geometrical constructions concerning lines and angles are given below.

Lines

To draw a straight line parallel to a given straight line at a given distance (Fig. 39):

AB is the given straight line and the line ST the required distance apart of the parallels. Using compasses, with centres A and B, and radius equal to ST, describe arcs L and M. Draw a line CD so that it just touches both arcs (i.e. the line is the common tangent to both arcs). CD is parallel to AB.

To draw a straight line parallel to a given straight line through a given point (Fig. 40):

AB is the given straight line and C is the given point. Along AB take any convenient point L as centre, and with radius LC, describe the arc CM, M being the point where the arc cuts AB. With C as centre and the same radius describe an arc LD, equal to CM. (The length equal to CM, along LD from L can be marked using dividers or compasses.) A straight line drawn through CD is parallel to AB.

An alternative method to the above is as follows (Fig. 41):

AB is the given straight line and C the given point as before. With centre C describe an arc to touch AB (i.e. so that AB becomes tangential to the curve) at M. Then take any convenient point L along AB and with the same radius describe an arc, and through C draw a straight line CD tangential to it. CD is parallel to AB.

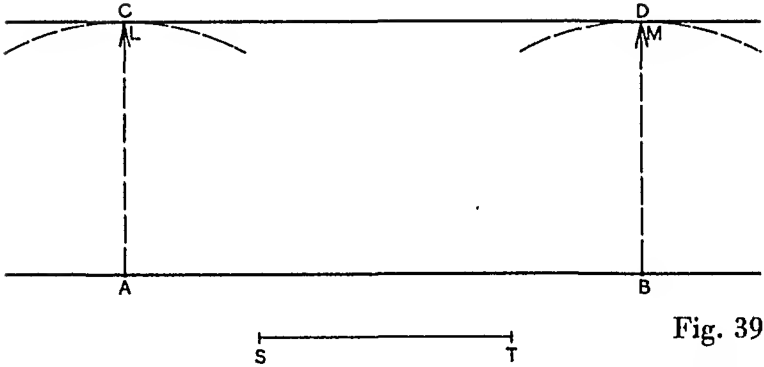


Fig. 39

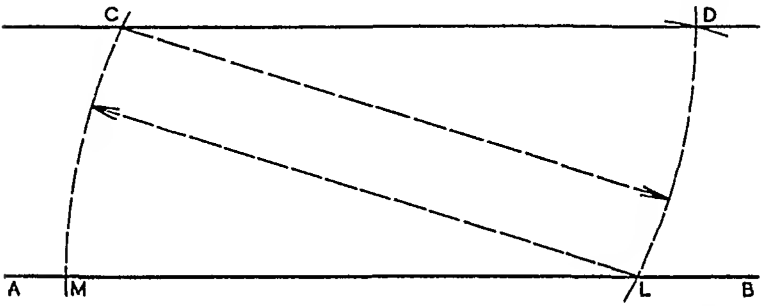


Fig. 40

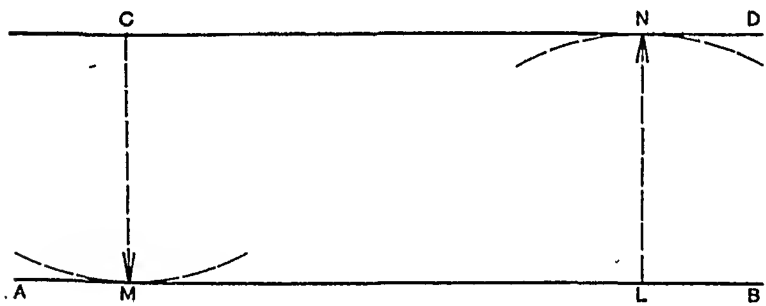
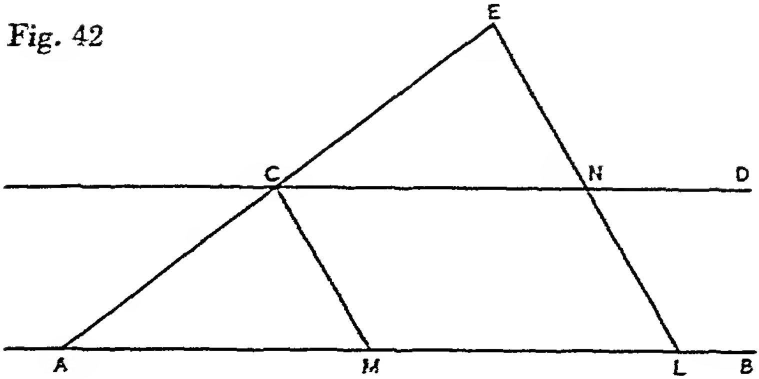


Fig. 41

Fig. 42



Another alternative method, which does not need compasses (Fig. 42):

AB is the line and *C* the point as before. Take any point *M* along *AB* and join to *C*. Draw a straight line from *A* through *C* and continue to *E* so that *CE* equals *AC*. *L* is a point in *AB* so that *ML* equals *AM*. Join *LE*. From *L* along *LE* measure *LN* equal to *MC*. A straight line *CD* drawn through *C* and *N* is parallel to *AB*.

Angles

✓ *To bisect a given angle* (Fig. 43):

CAB is the given angle. With vertex *A* as centre and any radius describe an arc to cut *AB* and *AC* at points *L* and *M* respectively. With centres *L* and *M* and any convenient radius describe arcs to intersect at *D*. A line drawn through *D* from *A* bisects (i.e. divides into two equal parts) the angle.

✓ *An alternative method which does not need compasses* (Fig. 44):

CAB is the given angle as before. Mark two convenient points *L* and *N* along *AB*, and two corresponding points *M* and *O* along *AC*, so that *AL* equals *AM* and *AN* equals *AO*. Join *L* to *O* and *N* to *M* so that *LO* and *NM* intersect at *P*. A line *AD* drawn through *P* bisects the angle.

✓ *To trisect (i.e. divide into three equal parts) a right-angle or the quadrant of a circle* (Fig. 45):

BAC is the right-angle, *AB* equals *AC*. With centre *A* and radius *AC* describe the arc *CB*, and with the same radius and centres *C* and *B* describe arcs to cut the arc *CB* at *L* and *M*. Lines joining *L* and *M* to *A* trisect the angle.

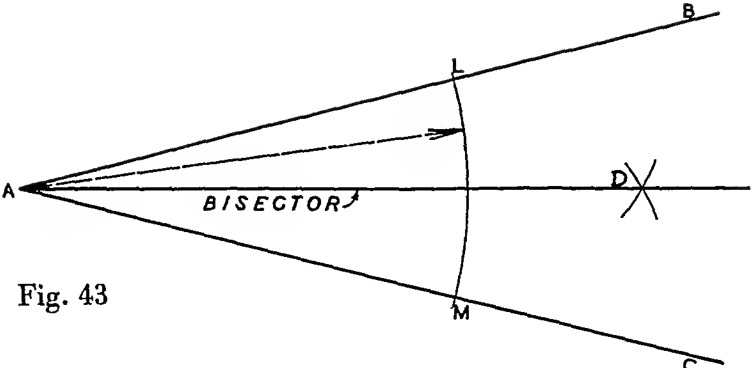


Fig. 43

BISECTION OF ANGLES

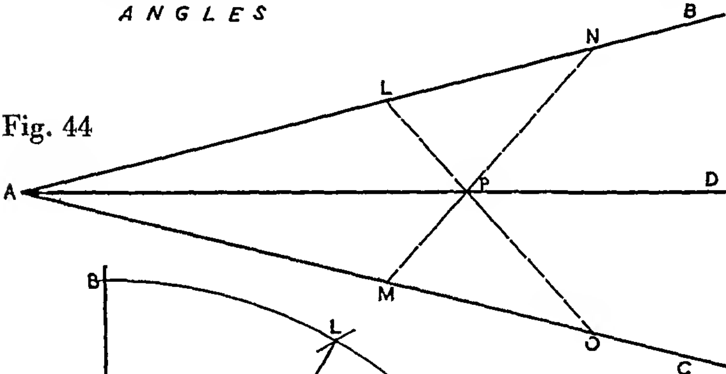


Fig. 44

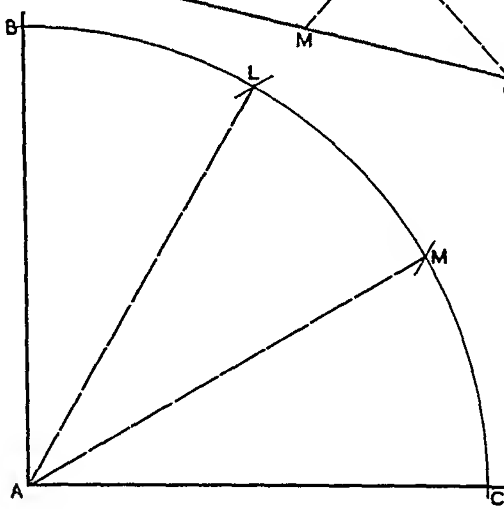


Fig. 45

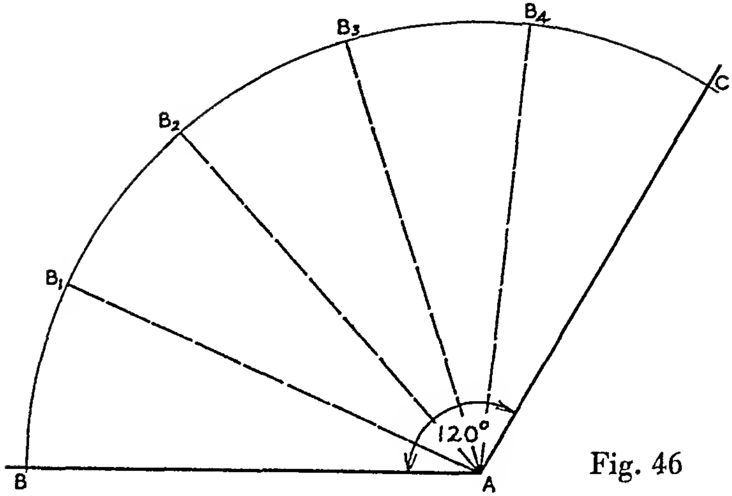


Fig. 46

To divide any given angle into a given number of equal parts (Fig. 46):

BAC is an angle of 120 degrees, to be divided into 5 equal angles. With the aid of a protractor, set off from *A* angles of 1/5 of 120 degrees, 2/5 of 120 degrees, etc. For accuracy check the chords *BB*1, *B*1, *B*2, etc., by means of dividers.

To erect a perpendicular to a given straight line at a given point on the line (Fig. 47):

AB is the given straight line, *C* the given point on it. With centre *C* and any convenient radius describe arcs cutting *AC* and *CB* at points *L* and *M*. With centres *L* and *M* and any radius greater than *CM* describe arcs to intersect at *D*. A straight line joining *D* to *C* is perpendicular to *AB*.

To draw a perpendicular to a given straight line at one end of the line (Fig. 48):

AB is the given line, and the perpendicular is to be drawn downwards from *B*. Take any convenient point *L* (below *AB*) as centre, and with radius *LB* describe an arc to cut *AB* at *M*. Join *ML* and produce this line to cut the circular arc in *N*. Join *NB* which is perpendicular to *AB*. (This construction is based on Euclid's theorem that if from the extremities of the diameter of a circle, lines be drawn to meet at a point on the circumference, these two lines will include a right-angle.)

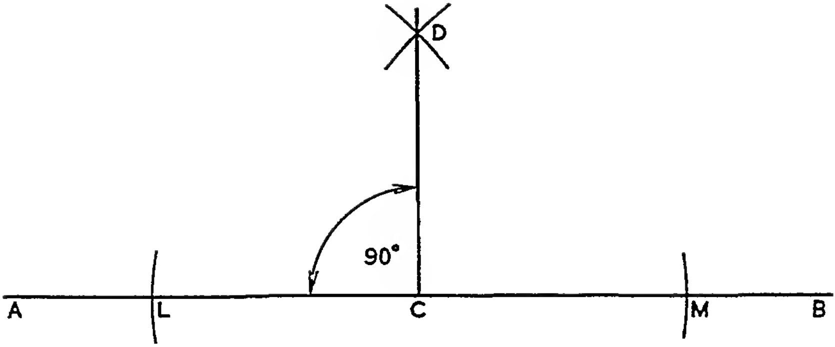


Fig. 47

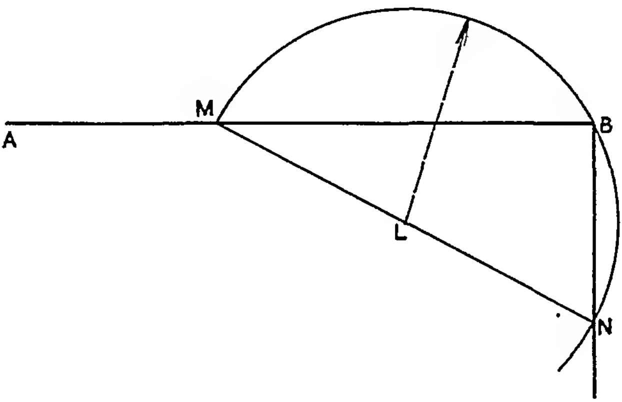


Fig. 48

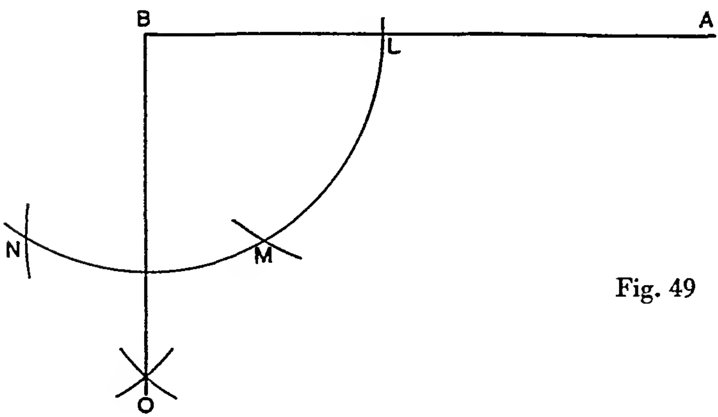


Fig. 49

An alternative method (Fig. 49):

With centre B and any convenient radius describe an arc to cut AB at L. With centre L and the same radius describe an arc to cut the first arc at M, and repeat with centre M to find N. Bisect angle MBN as previously described (Fig. 43), and the bisector, BO, is perpendicular to AB.

To set out a right-angle using a chain or tape:

Fig. 50 shows the method as it might be used for setting out on the site the centre lines of foundation trenches for a rectangular building. A wooden peg is driven into the ground at A, the point of intersection of the centre lines of two trenches. A distance of, say, 6′ 0″ is measured in the known direction of one trench, and another peg is driven into the ground at B. A tape is then passed round the pegs, and pulled tight in such a way that a triangle is formed with sides 6′ 0″, 8′ 0″ and 10′ 0″ long, thus giving point C, and angle BAC equal to 90 degrees.

Fig. 51 gives a sketch showing how profiles would be arranged together with the builder's line before the trench could be excavated.

The basis of this method, in which any suitable measurements can be taken, is the well-known "Theorem of Pythagoras", which proves that, in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the sides containing the right-angle (see Fig. 52).

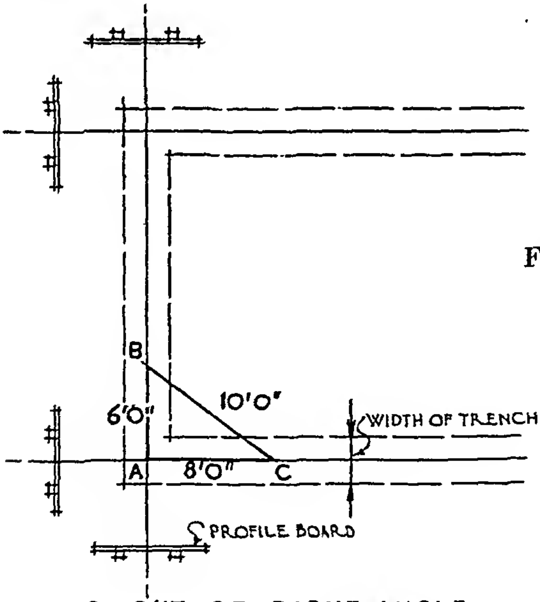
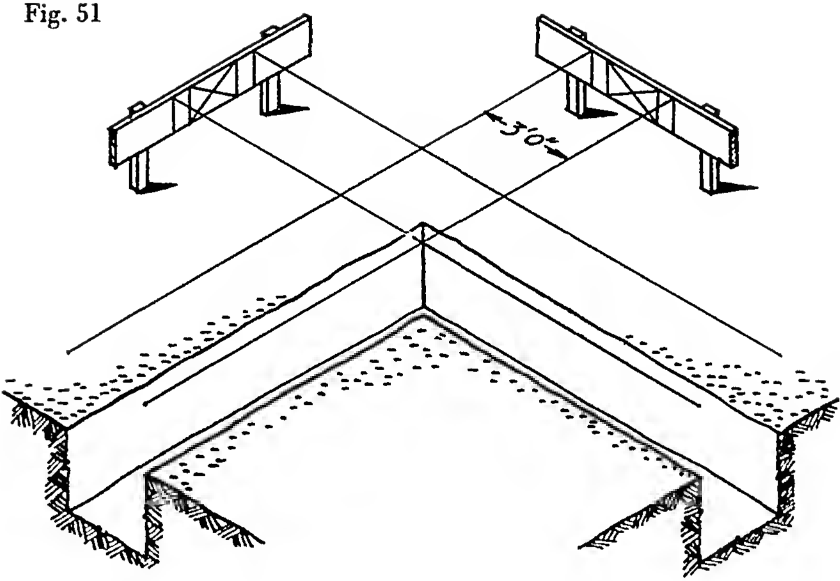


Fig. 50

SETTING OUT OF RIGHT-ANGLE

Fig. 51


3'0"

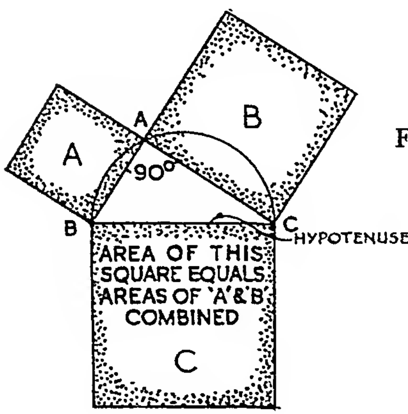

A
B
A
90°
B
C
HYPOTENUSE
AREA OF THIS SQUARE EQUALS AREAS OF 'A' & B' COMBINED
C

Fig. 52

Triangles

Definitions. A triangle is a closed figure formed by three straight lines, and containing three angles (see also Chapter II).

The side which is horizontal, or most nearly horizontal, is the one generally termed the *base*, although for purposes of practical surveying the longest line is usually the base line. The two adjacent sides are termed boundaries. *Base angles* are those at each end of the base, the remaining angle is the *vertical angle* or *vertex.*

The *altitude* or *height* of a triangle is the perpendicular distance from the vertex to the base.

A *median* is a line drawn from any angle to the middle of the opposite side.

The *perimeter* of a triangle is the sum of the lengths of its three sides.

The following geometrical properties of triangles should be noted:

(*a*) A triangle in a circle is a right-angled triangle when one side is a diameter of the circle and the vertex lies on the circumference (see Fig. 48).

(*b*) Pythagoras' theorem states that the square on the hypotenuse (longest side) of a right-angled triangle equals the sum of the squares on the other two sides (see Fig. 52).

(*c*) The sum of the three angles of any triangle equals 180 degrees.

(*d*) The longest side in any triangle always lies opposite the largest angle (see Fig. 48).

(*e*) Similar triangles are triangles in which the three angles of the one are each equal to the corresponding angle of the other —that is, they are equiangular.

Right-angled triangle: one angle of 90 degrees, and two others together equal to 90 degrees.

Equilateral triangle: three equal sides and three equal angles, each 60 degrees.

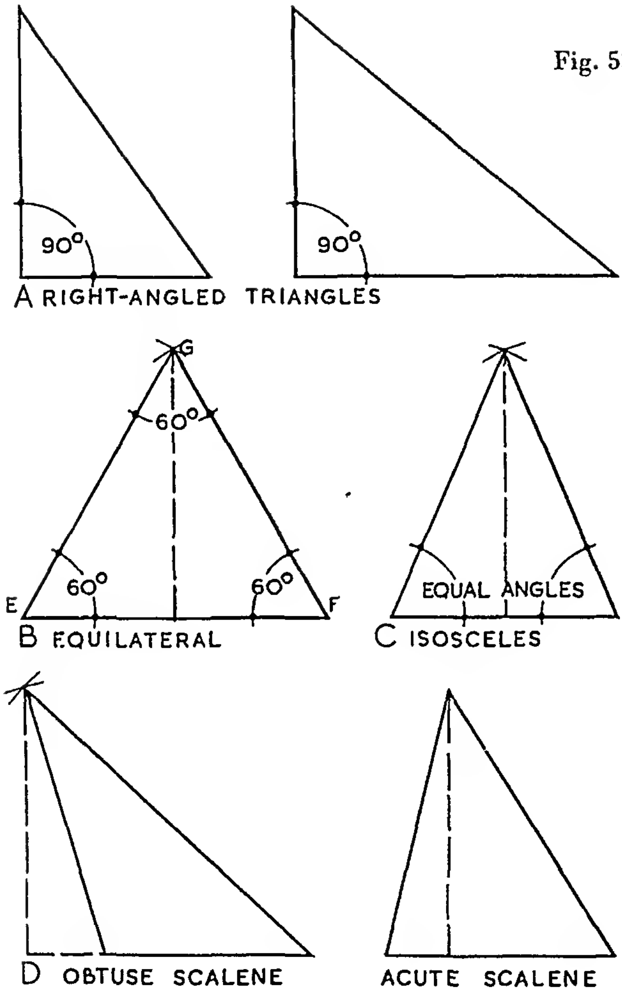
Isosceles triangle: two equal angles and two equal sides.

Scalene triangle: three unequal angles and three unequal sides.

For illustrations of the foregoing see Fig. 53.

TRIANGLES

Fig. 53



Examples

To construct a right-angled triangle with sides adjacent to the right-angle of given length (Fig. 54):

The lengths of the adjacent sides are a and b. Draw AB equal to a, and at B erect a perpendicular of length equal to b. Join B to C. ACB is the required triangle of which BC is the hypotenuse.

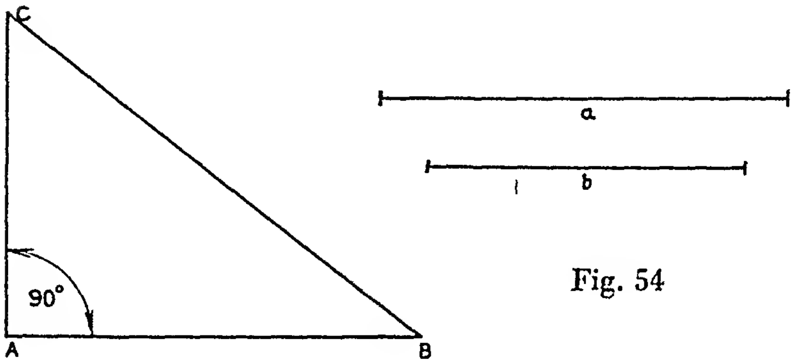


Fig. 54

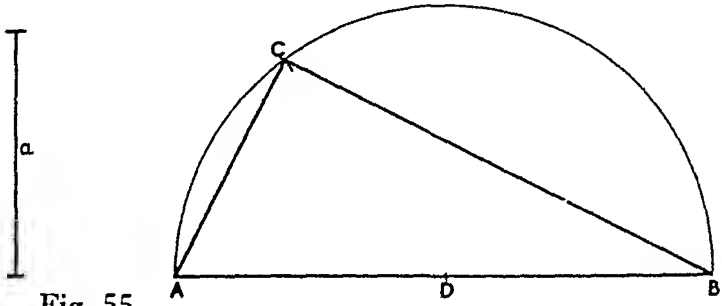


Fig. 55

To construct a right-angled triangle, the hypotenuse and one other side (a), *which must be less than the hypotenuse, being given* (Fig. 55):

Draw AB, the given hypotenuse, and bisect it to find point D. With centre D and radius DB, describe an arc. With centre A and radius equal to a, describe an arc to intersect the first arc at C. Join C to A and B. ACB is the required triangle, and CB is the third side.

To construct an isosceles triangle, given the length of the two equal sides and the size of the two equal angles (Fig. 56):

Let ST be the given sides and θ the equal angles. Draw AB equal to ST. With A as centre and radius AB, draw the arc BC. At B set off the angle ABD equal to θ, and where this line cuts the arc gives the point D. Join A to D. Then ABD is the required triangle.

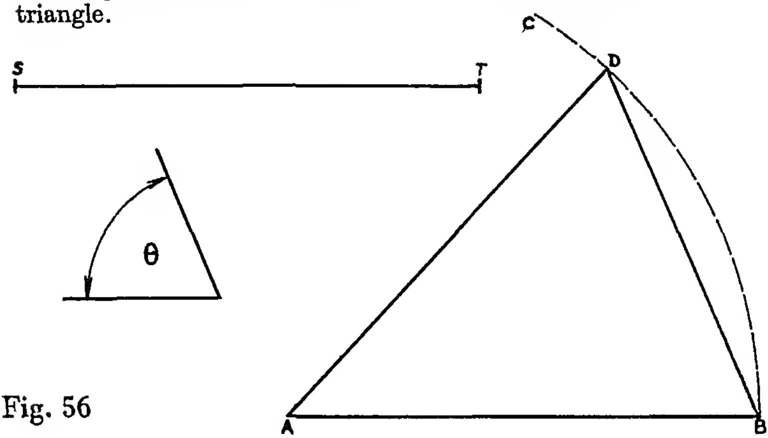


Fig. 56

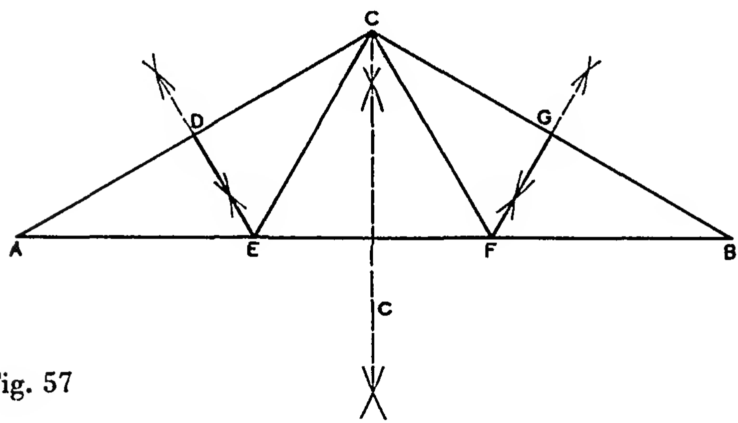


Fig. 57

To draw the main lines of a standard steel roof-truss (Fig. 57):

Draw AB to represent the span of the truss, and the perpendicular bisector, CC of AB. Along the bisector mark off the required height of the truss from the line AB. Join AC and BC. On AC erect the perpendicular bisector, and continue this line, cutting AC at D, to cut AB at E. Join EC. Similarly, construct GF and FC.

To draw the outline of a simple roof-truss which forms an isosceles triangle and which contains four equal isosceles triangles formed by the collar and struts (Fig. 58):

Draw AB equal to the span of the roof. With centres A and B, and radius equal to the length of the sides of the truss, describe arcs to intersect at C. Join A to C and B to C. ABC is the outline of the truss. Bisect AB, AC and BC, to find the points D. E and F. Join E to D, D to F and F to E, and the lines of the members of the truss are complete. AEF, ECD, FED, FDB are equal isosceles triangles.

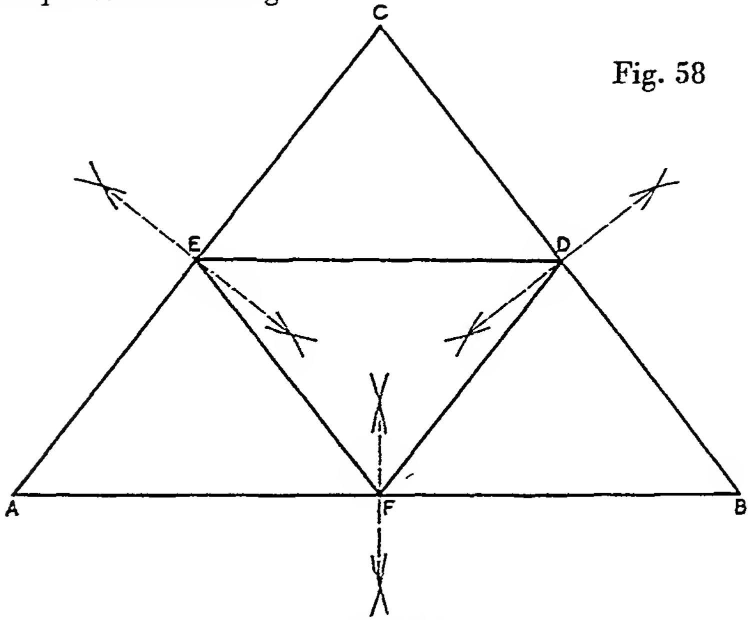


Fig. 58

To draw the main lines of another type of truss (Fig. 59):

Draw a semicircle with diameter representing the required span AB. With centres A and B, and the same radius, describe arcs, to cut the semicircle at C and D. Join A to C, C to D, D to B, A to D and B to C, and from C and D draw perpendiculars to AD and CB respectively. (E is midway along AD, and F is midway along CB.)

Scalene Triangles

To draw a triangle, the base (a) *and the sides* (b) *and* (c) *being given, and to find its altitude* (Fig. 60):

Draw the baseline AB equal to a. With centre A and radius equal to b, draw an arc to intersect at C an arc described with centre B and radius equal to c. Join A and B to C. From C draw a perpendicular CD to AB. CD is the altitude.

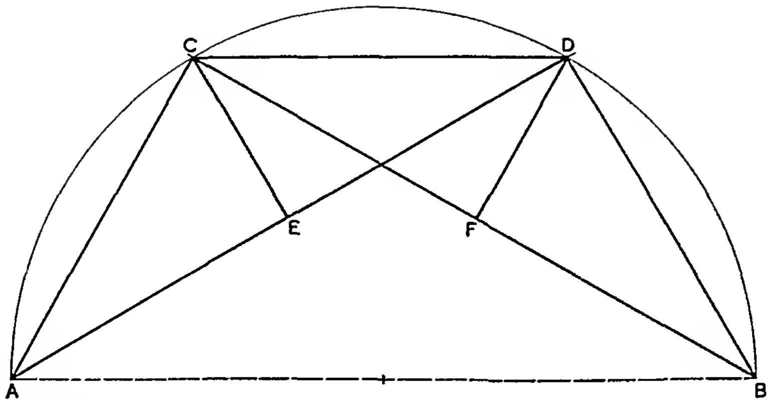


Fig. 59

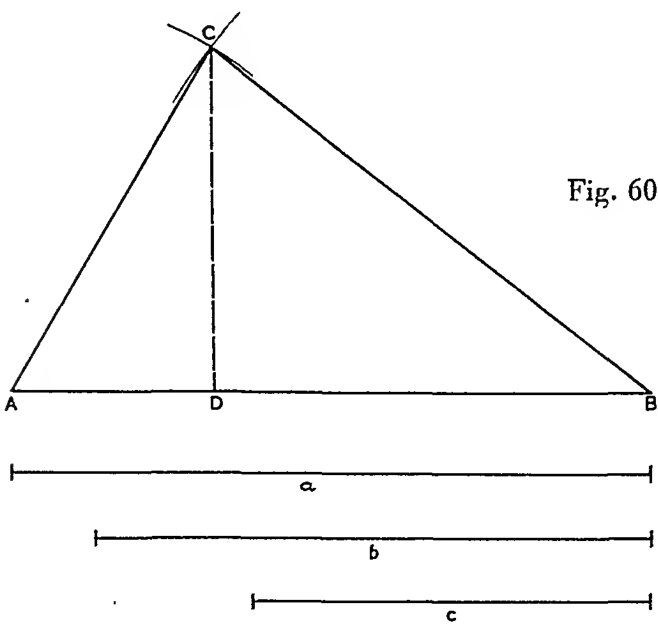


Fig. 60

To construct a triangle, the base angles θ and ϕ and the altitude (a) being given (Fig. 61):

Construct a triangle ACD, making angle ACD equal θ, and angle ADC equal ϕ. Draw a line EF parallel to CD, at a distance equal to the required altitude. Produce CA to cut this line at $A1$. At $A1$ draw $A1D1$ parallel to AD to meet CD produced at $D1$. $A1D1C$ is the required triangle.

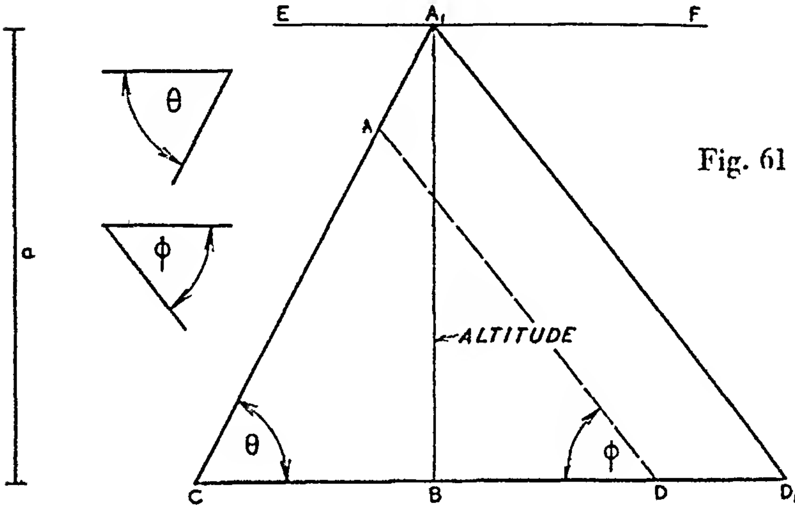


Fig. 61

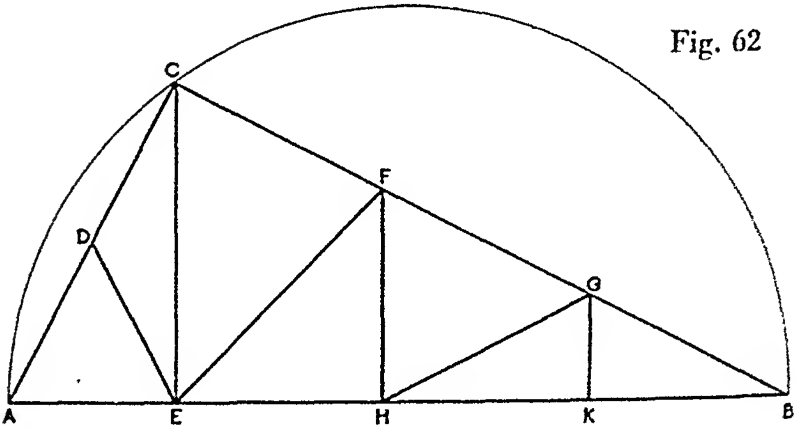


Fig. 62

To draw the main lines of a north-light roof-truss, the span and the base angles being known. (The base angles give the "pitch" of the roof.) (Fig. 62):

The outline of the truss is drawn by the method previously described and illustrated in Fig. 61. The positions of F and G are found by dividing CB into three equal units; D is midway along CA, CE, FH and GK are perpendicular to AB.

PRACTICAL APPLICATION

The Plane Table

This is a simple instrument for making approximate surveys, and is used in connection with military sketches. It consists of a small drawing-board fixed to a tripod so that a sighting rule or "alidade" can be levelled and plumbed in any position. A sheet of drawing paper is pinned to the board and by means of the "alidade", the direction of any object can be marked on the paper. Fig. 63 shows a sketch of the plane table with ranging poles staked into the ground at various points, C D E F, known as "stations". The tripod is plumb over a wood peg. A, temporarily replacing a ranging pole. Sight lines are taken to the stations C D E F and B and ruled on the paper, Fig. 64. The distance AB, the base line, is measured and drawn to a suitable scale, say 88′ 0″ to 1″, on the paper in the correct positions. The plane table, etc., is then moved and placed plumb over station point B. The sight lines are taken as before and the directions drawn on the paper. The result is the formation of a series of triangles on the paper, all with the common base, AB, and with apices giving to scale the positions of C D E F, the distances of which can be measured. The acreage of the land surveyed can also be found by simple calculation.

The Box Sextant

Another method of surveying land is that in which the box sextant is used. A full description is not possible here, but it is about 3″ in diameter, sometimes with a telescope attached. It is fitted with a scale and vernier curved to a flat arc, radius 2″, of about 150 degrees; the scale commences at 5 degrees below zero and is divided into degrees and half degrees. The vernier is the length of 29 of these degrees and is divided into 30 equal parts, thus enabling angles to be read in degrees and minutes—the reading being made with the help of a magnifying glass. With some models it is only possible to read angles up to 90 degrees, and consequently intermediate settings are necessary if greater angles are required.

Traverse Survey using the Box Sextant

The use of the box sextant is illustrated in Fig. 65. A stream forms the boundary of a field and it is required to plot its winding course. A number of ranging poles are placed at convenient positions as near as is practicable to the edge of the stream with the object of making a series of triangles which will tie each other forming a traverse survey. Commencing at station point *A* and proceeding in the direction indicated by the arrows the box sextant

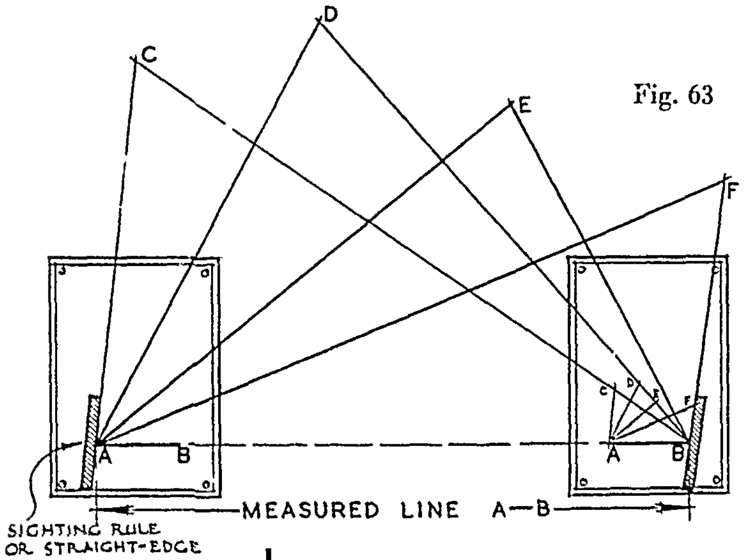


Fig. 63

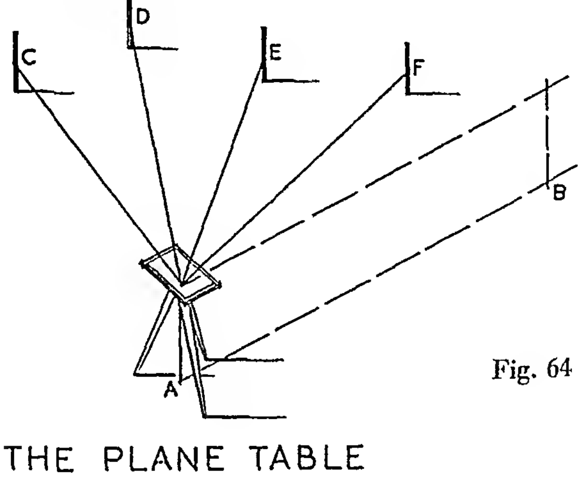


Fig. 64

THE PLANE TABLE

is held close to the ranging poles and the readings of the angles indicated are made and booked, afterwards being drawn on paper. The box sextant is not accurate for trigonometrical surveys, but it is a useful check on chain surveys. The theodolite is the most accurate instrument.

Traverse Survey using Chain

Fig. 66 shows the method of making a simple traverse survey using the "Gunter's" Chain (100 links) or an "Engineers" Chain

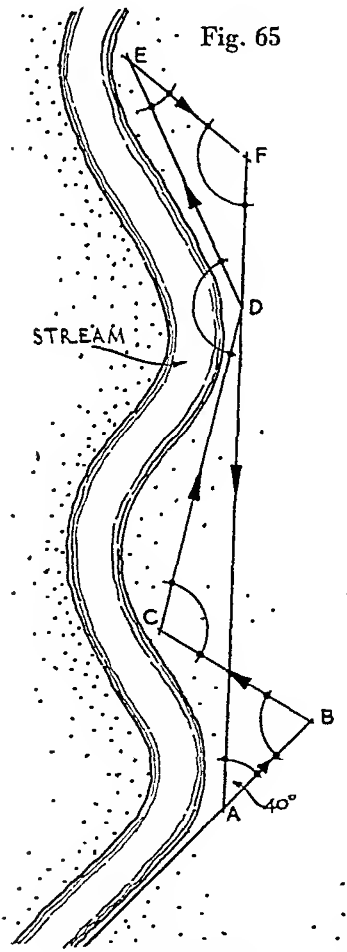
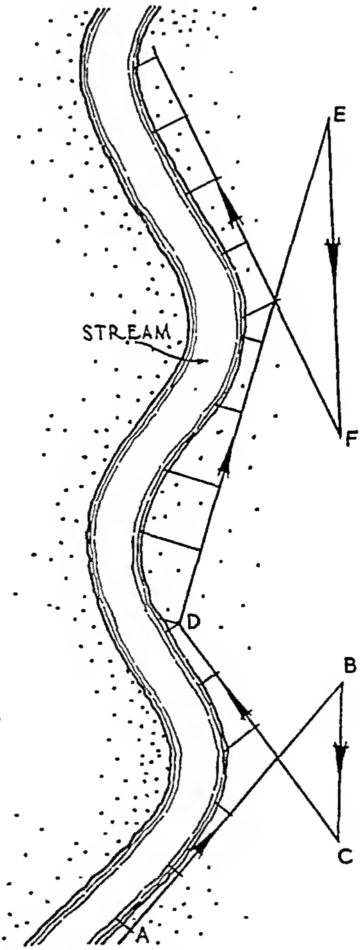


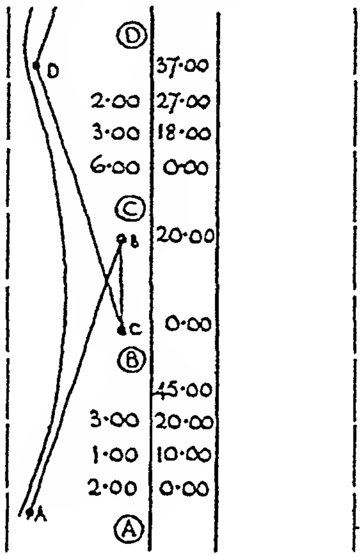
Fig. 65



TRAVERSE SURVEY

Fig. 66

(in feet). Ranging poles are placed as before. Commencing at station A, running dimensions are taken in the direction indicated by the arrows and booked in the appropriate column of a field book. Offset measurements are also taken at frequent intervals at right-angles to the chain and from it to the edge of the stream. These measurements are likewise recorded in the field book. Fig. 67 shows part of such recordings as set out in the field book.



FIELD BOOK

Fig. 67

The Clinometer

Instruments for measuring slopes are known as clinometers. The best known is the theodolite which, as previously mentioned, is also used for measuring flat angles.

To find the height of a tall building (Fig. 68) :

At some convenient measured distance from the building the instrument is set up. In front of it at, say, 10′ 0″ distance, at EF, an upright stake is firmly planted in the ground. By sighting through the telescope of the clinometer to the top of the building, a mark can be made on the side of the stake where the sight-line cuts it. If the height of the telescope above the ground is 5′ 0″ and the height of the mark on the stake is 8′ 0″, then the rise of the sight line is 3′ 0″ in 10′ 0″, and therefore, if the distance of the instrument from the building is 300′ 0″, the height of the building to 300′ 0″ will be as 3 is to 10, that is 3/10 of 300, or 90′ 0″, plus the height of the instrument, 5′ 0″, making 95′ 0″ in all.

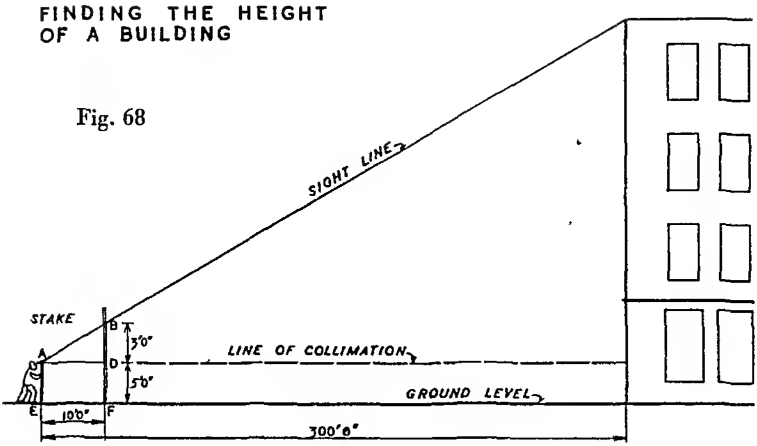


Fig. 68

Measurement Through Obstructions

To complete a measurement when the line of measurement is obstructed (Fig. 69):

The line of measurement being obstructed by a building, set out at *A*, a convenient point close to the building, a right-angled triangle with base of sufficient length to clear the obstruction. At *B* set out another right-angled triangle, and repeat at *C* and *D*, the distance *AB* being equal to *CD*. The length of *BC* added to the lengths of the main line complete the required measurement.

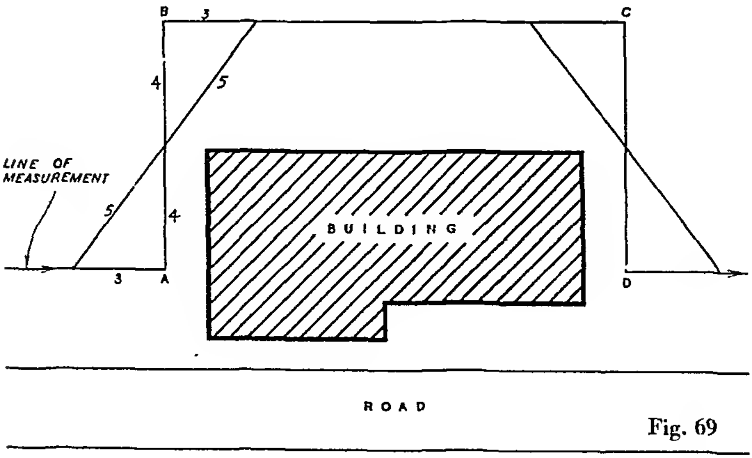


Fig. 69

To measure the width of a stream (Fig. 70):

Set out AB at right-angles to the stream, with B on the edge of the bank, to measure, say, 10′ 0″. From A set out AC at right-angles to AB and also 10′ 0″ in length. By sighting along AB, a point D on the edge of the opposite bank can be found. From D set out DE at right-angles to D, the position of E being found by sighting along CB. The distance DB equals DE, the required width of the stream (see also Fig. 71).

To measure a distance at an angle across a stream (Fig. 72):

As close as possible to a bank of the stream, set out ADB in a straight line so that AD equals DB at some convenient measurement. Set out AC at right-angles to AD, and BF at right-angles to BD. (If instruments are not used, a perpendicular BF would have to be set out first, so that by sighting along FB, the direction of BE can be found.) By sighting through D, so that C and E are in the same straight line, the distance CD, which can be measured, equals DE, the required distance.

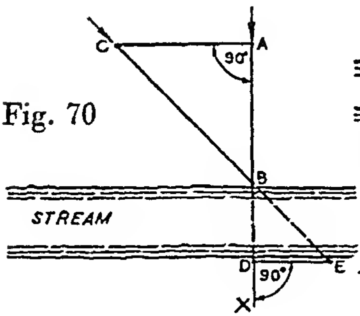


Fig. 70

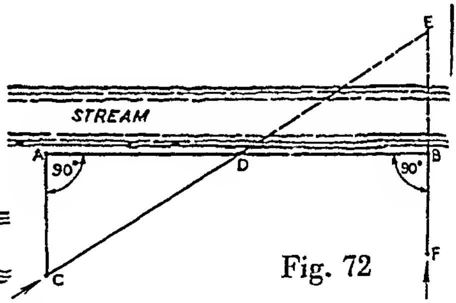


Fig. 72

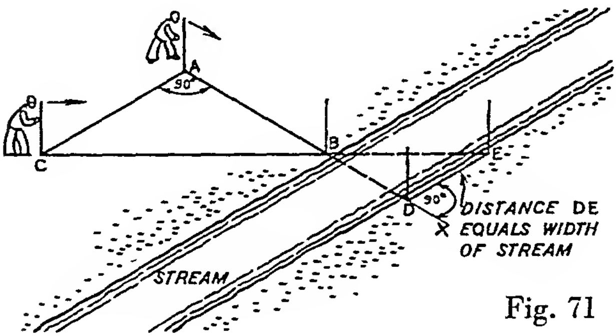


Fig. 71

In actual practice, perpendiculars or lines at right-angles are set out with great accuracy by means of instruments. The simplest is the cross-staff, the use of which is illustrated by Figs. 73 and 74.

In measuring distances over undulating ground, or on steeply sloping ground, for all but the most important surveys, what is

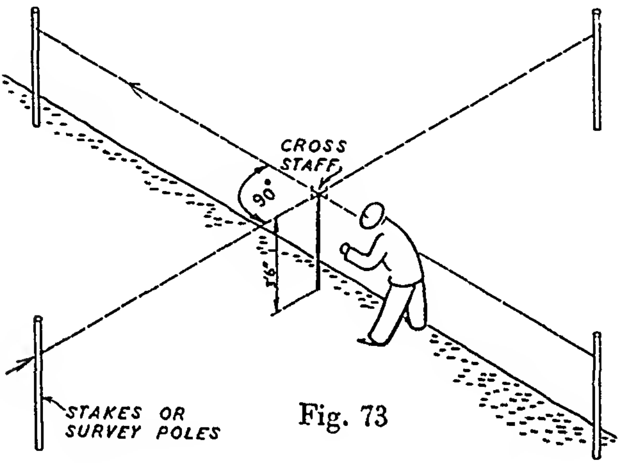


Fig. 73

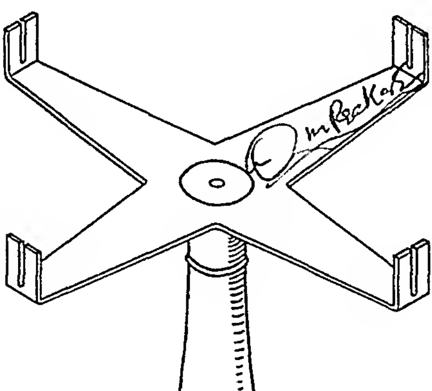


Fig. 74

known as *step chaining* is employed—Fig. 75. If, for example, the rise of the ground is about 1 in 5, stakes are planted in the direction of the required measurement, at regular intervals of, say, 50 links. The chain or tape is then held tightly against the base of the second stake, *B*, and pulled taut and as horizontal as possible to the first stake, *A*, which is kept vertical with the help of a plumb rule. The procedure is repeated between each pair of stakes, so that an accurate horizontal measurement of the distance can be obtained.

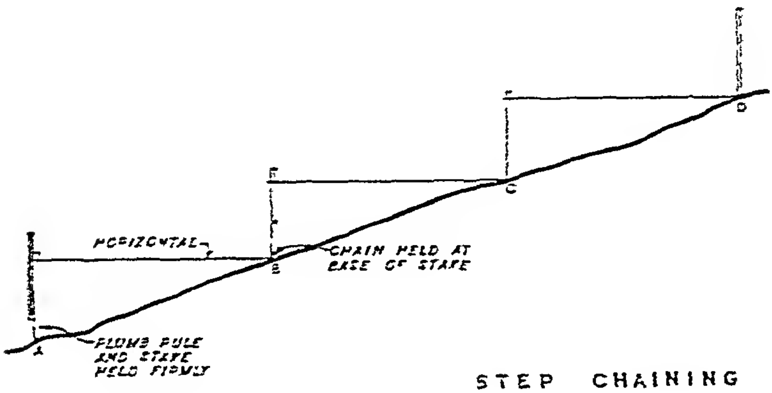


Fig. 75

CHAPTER V

QUADRILATERALS

SETTING OUT OF VARIOUS TYPES; APPLICATION TO LAND SURVEYING

DEFINITIONS and illustrations of quadrilaterals have already been given in Chapter II. (Note—the sum of the interior angles of any quadrilateral equals 360 degrees.)

The Square

To construct a square, the length of the sides being given (Fig. 76):

Draw a straight line AB equal to the given length. With centres A and B and radius AB, describe arcs to cut perpendiculars erected from A and B in C and D respectively. Join A to C, C to D and D to B. $ACDB$ is the required square.

An alternative method (Fig. 77):

Draw AB as before. From A draw a line at 45 degrees to AB, to cut a perpendicular erected from B at D. Similarly, draw a line at 45 degrees from B to cut a perpendicular from A at C. Join A to C, C to D and D to B. $ACDB$ is the required square.

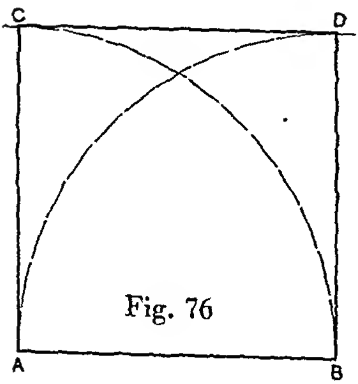


Fig. 76

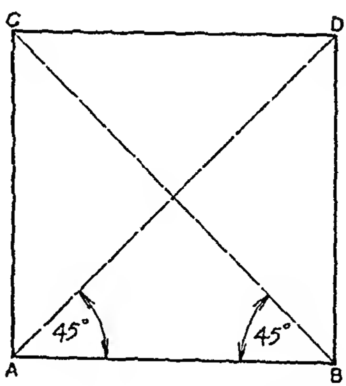


Fig. 77

To construct a square, the diagonal being given (Fig. 78):

Draw a straight line AB equal to the length of the diagonal. Bisect it to find O. With O as centre and radius OA describe a circle to find points C and D on the perpendicular bisector of AB. Join A to C, C to B, B to D and D to A. $ACBD$ is the required square.

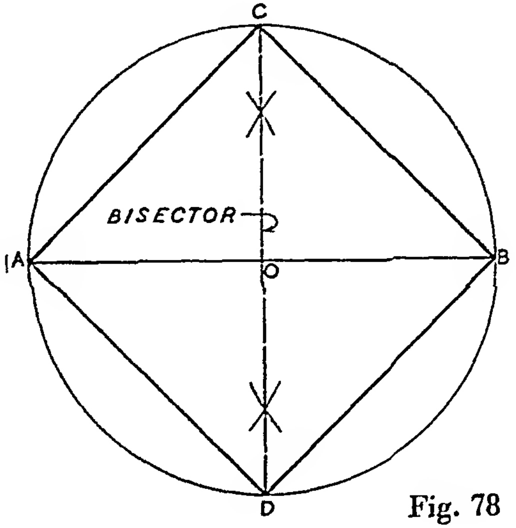


Fig. 78

The Rectangle

To construct a rectangle, the base and height being given (Fig. 79):

Draw AB equal to the length of the base. From A and B erect perpendiculars and along them mark the length of the height to find points C and D. Join A to C, C to D and D to B. $ACDB$ is the required rectangle.

To construct a rectangle, the diagonal and the length of one side being given (Fig. 80):

Draw AB equal to the diagonal and bisect it to find O. With centre O and radius OA describe a circle. With centre A and radius equal to the length of the given side, describe an arc to cut the circumference of the circle in C. From B describe a similar arc to find D. Join A to C, C to B, B to D and D to A. $ACBD$ is the required rectangle.

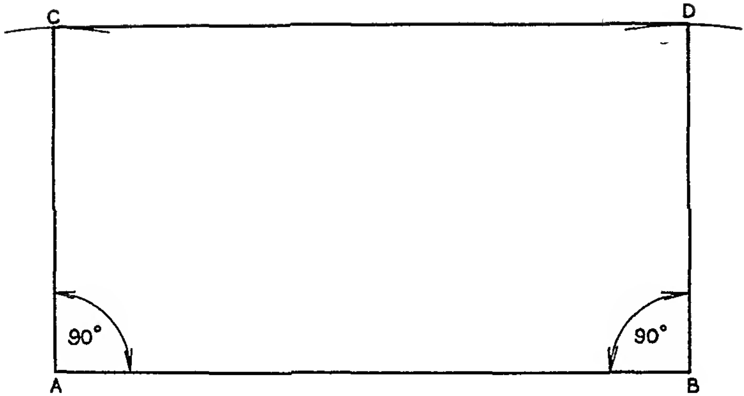


Fig. 79

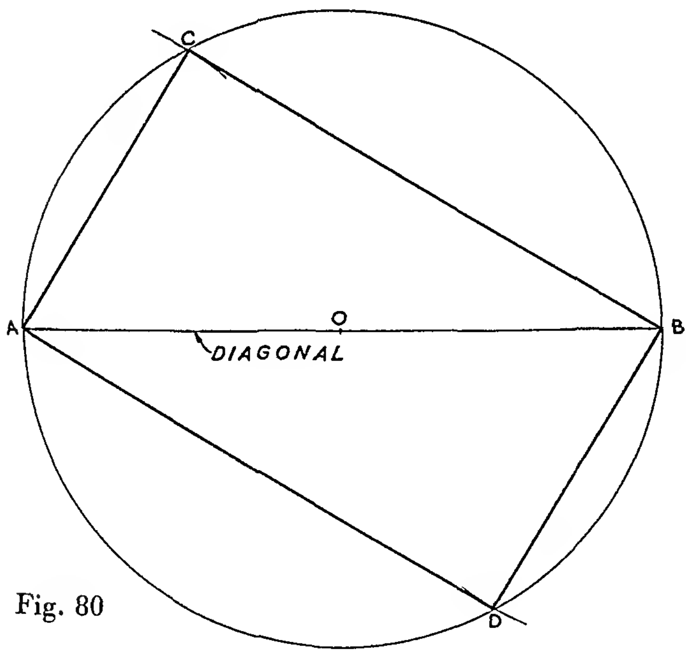


Fig. 80

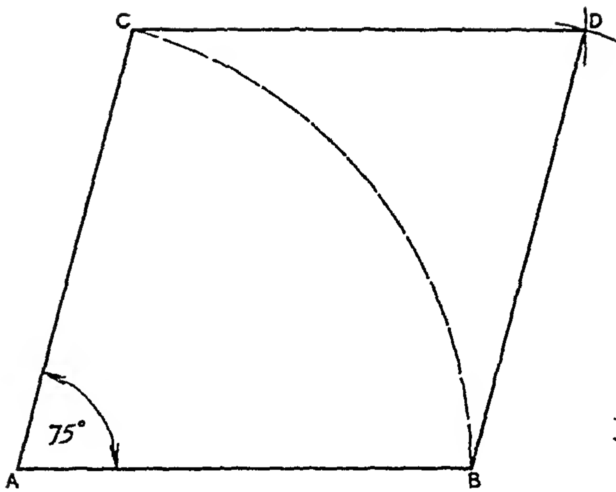


Fig. 81

The Rhombus

To construct a rhombus, the length of the base and the base angle (75 degrees) being given (Fig. 81):

Draw AB equal to the base. From A, using protractor or adjustable set-square, set off a line at 75 degrees to AB, and with centre A and radius AB describe an arc to cut this line at C. With centres C and B and the same radius, describe arcs to intersect at D. Join A to C, C to D and D to B. $ACDB$ is the required rhombus.

To construct a rhombus, the lengths of the diagonals being given (Fig. 82):

Draw AB equal to one of the diagonals, and the perpendicular bisector COD, intersecting AB at O. Make OC equal to OD, equal to half the other diagonal. Then the rhombus is obtained by joining A to C, C to B, B to D and D to A. (In a rhombus the diagonals bisect one another at right-angles.)

To construct a quadrilateral similar to a given quadrilateral, (a) the length of one of the diagonals, or (b) the length of one of the sides, being known (Fig. 83):

(*a*) $ABCD$ is the given quadrilateral, and a similar quadrilateral is required, having the diagonal of length 3 inches. Join AC and produce it to C_1, making AC_1 equal 3 inches. From C_1 draw C_1D_1 parallel to CD, meeting AD produced in D_1, and draw C_1B_1 parallel to CB meeting AB produced in B_1. Then $AB_1C_1D_1$ is the required quadrilateral.

(*b*) $ABCD$ is the given quadrilateral, and a similar quadrilateral is required, having the side corresponding to AB $2\frac{3}{4}$ inches long. Produce AB to B_2, so that AB_2 equals $2\frac{3}{4}$ inches. From B_2 draw B_2C_2 parallel to BC, meeting AC produced in C_2. From C_2 draw C_2D_2 parallel to CD meeting AD produced in D_2. Then $AB_2C_2D_2$ is the required quadrilateral. (Note—if AB_2 is less than AB, the required quadrilateral will come within the given quadrilateral.)

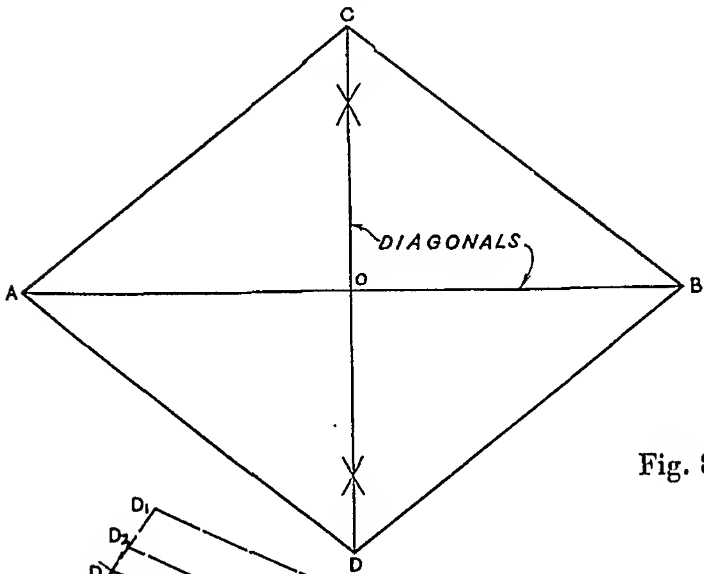


Fig. 82

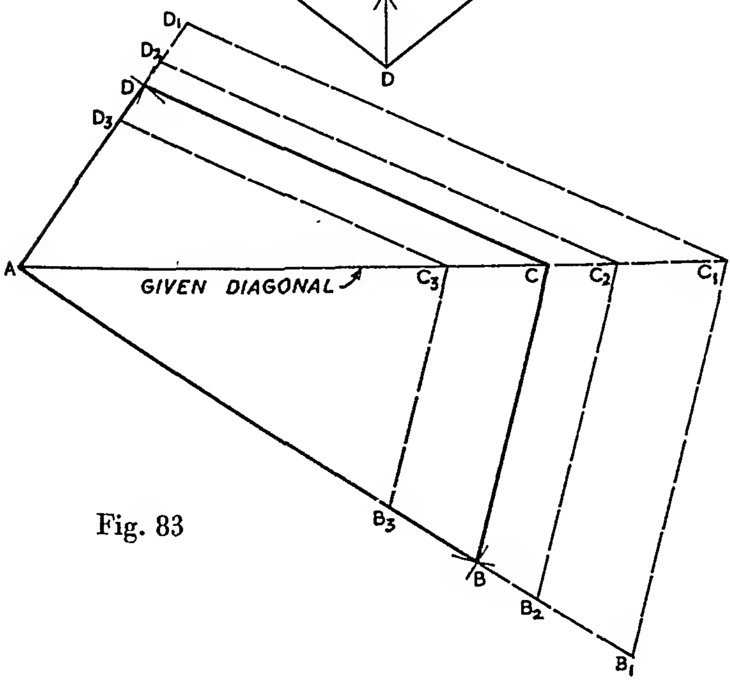
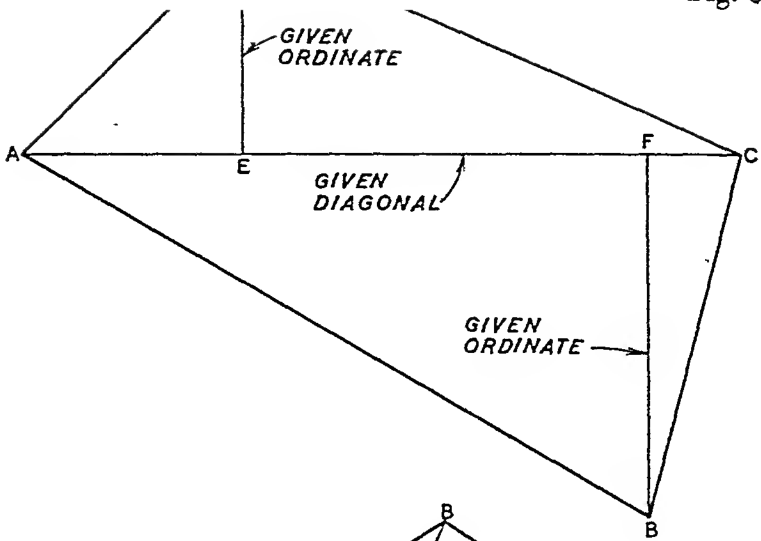


Fig. 83

Fig. 84


GIVEN
ORDINATE
A
E
F
C
GIVEN
DIAGONAL
GIVEN
ORDINATE
B

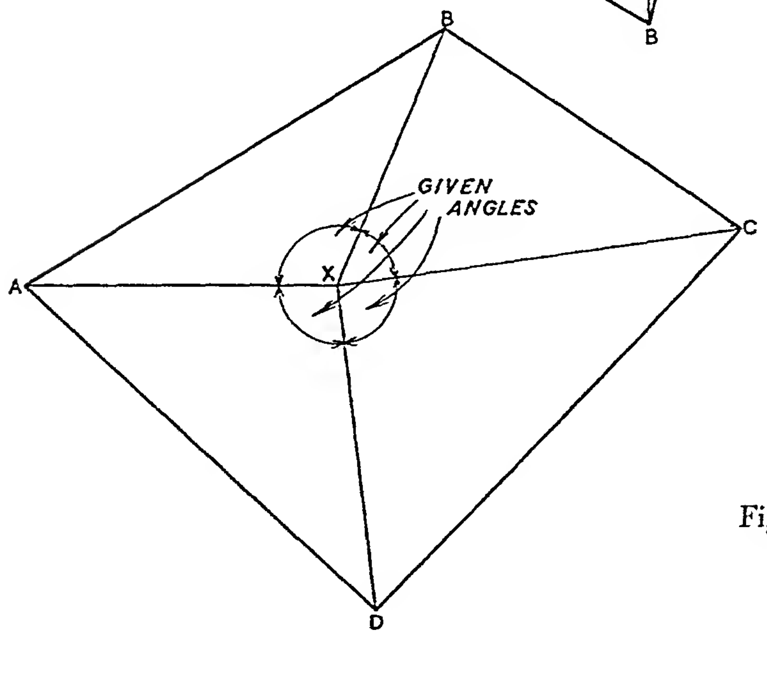

B
GIVEN
ANGLES
C
A
X
D

Fig. 85

To construct a quadrilateral, one diagonal, and the position and lengths of offsets or ordinates being given (Fig. 84):

Draw AC equal to the given diagonal. From the given position of the ordinates E and F, draw perpendiculars on either side of AC. Along these perpendiculars mark points D and B so that ED and FB respectively equal the lengths of the given ordinates. Join A to D, D to C, C to B and B to A. $ADCB$ is the required quadrilateral.

To construct a quadrilateral, the lengths of the sides (AB, BC, CD, DA), *a point* (X) *within the figure, the distance of this point from one corner* (AX), *and angles formed at this point by lines drawn from the corners of the figure, being given* (Fig. 85):

Draw AX of given length. At X, using a protractor or adjustable set-square, set off the given angles and draw corresponding lines from X. With centre A and radius equal to AB, describe an arc to cut the appropriate line from X at B. Join A to B. With centre B and radius BC similarly describe an arc to find point C, and so on, repeating the procedure to find point D. $ABCD$ is the required quadrilateral.

Use of Quadrilaterals in Surveying

Fig. 86 shows a field having a boundary of four sides. To survey this field ranging poles are staked at the corners, A, B, C and D. The straight distances indicated by the arrows are measured, using the chain, and diagonal ties or check measures are also taken. Offsets are taken to the boundary at regular intervals as explained in Chapter VI.

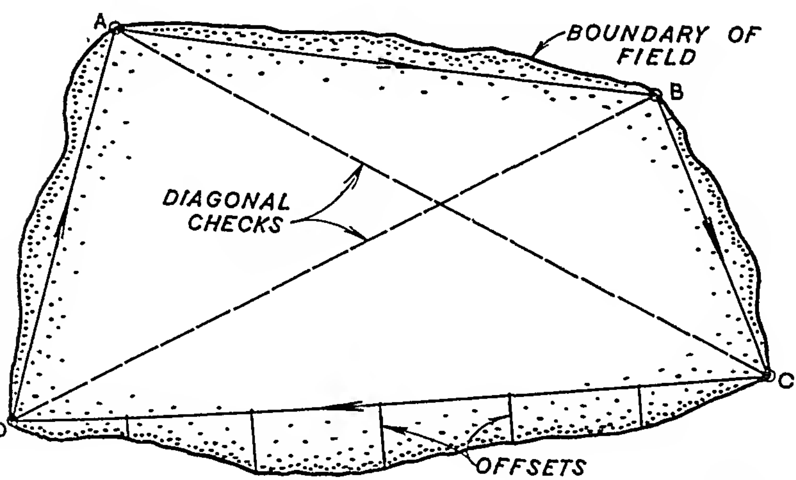


Fig. 86

CHAPTER VI

POLYGONS

SETTING OUT OF REGULAR AND IRREGULAR TYPES; APPLICATIONS IN LAND SURVEYING

Polygons

See "Definitions" (Chapter II).

Polygons are named according to the number of sides and angles. Shown in Fig. 87 are:

The *pentagon*, which has five sides.
The *hexagon*, which has six sides.
The *heptagon*, which has seven sides.
The *octagon*, which has eight sides.
The *nonagon*, which has nine sides.
The *decagon*, which has ten sides.
The *undecagon*, which has eleven sides.
The *duodecagon*, which has twelve sides.

The hexagon, octagon and duodecagon are most commonly used in building forms.

It is useful to remember that the sum of the interior angles of any polygon is equal to twice as many right-angles as the polygon has sides, minus the four right-angles at the centre, e.g. the degrees at the interior angles of an octagon are calculated as follows:

$$\frac{(2 \times 8) - 4}{8} \times 90 = 135 \text{ degs.}$$

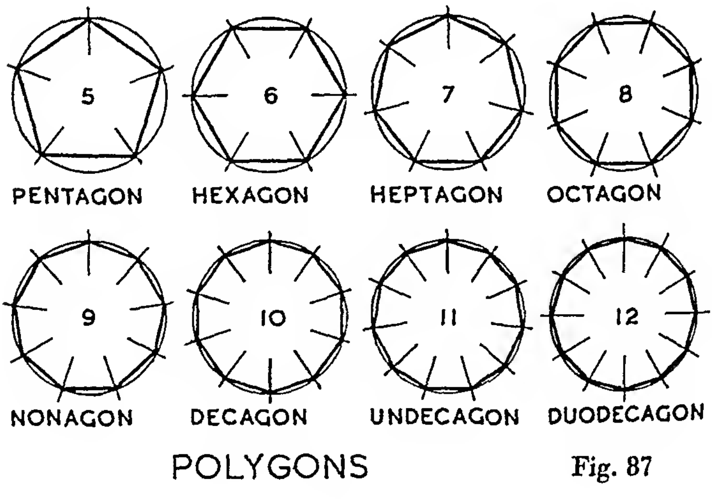


Fig. 87

There are many ways of drawing polygons for practical purposes. A mathematically correct method of drawing a regular polygon is illustrated by the following example:

To draw a regular pentagon with given length of side (Fig. 88): Draw AB equal in length to the given side. With centre A and radius AB draw a semicircle. Divide the arc of the semicircle into five equal parts, and index the points of division from left to right—1, 2, 3, 4. From A draw a line to point 2 (this line $A2$ is another side of the pentagon). To complete the figure, bisect $A2$ and AB. With O, the point of intersection of the bisectors, as centre, and radius OA, draw a circle (which will pass through 2 and B). This circle contains the required figure, which is found as indicated in Fig. 88. A regular heptagon can be constructed in a similar way, the semicircle being divided into 7 equal parts, and the line $A2$ being drawn as before.

The same method can be used to draw a regular polygon when the required length of a diagonal is given, as follows (see Chapter II, Fig. 13, for definition of "Diagonal"):

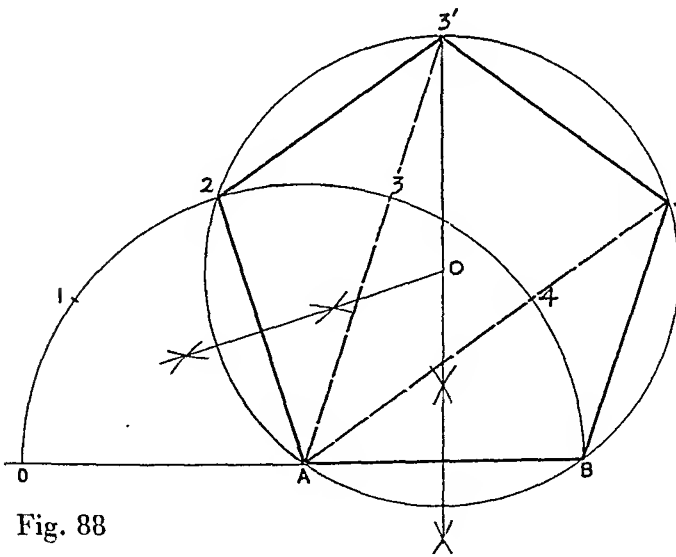


Fig. 88

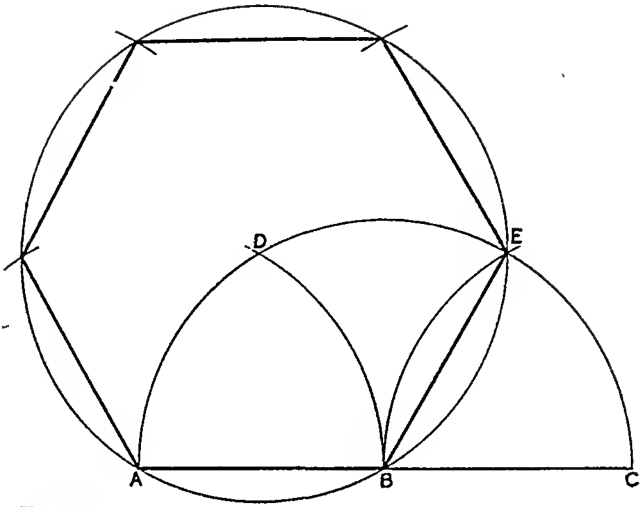


Fig. 90

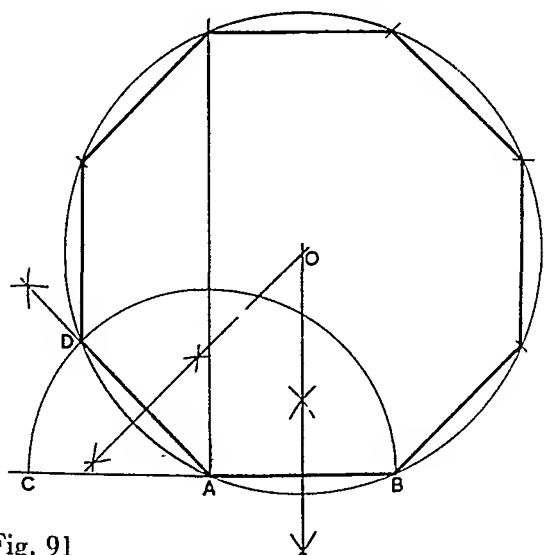


Fig. 91

To construct a regular hexagon with given length of side (Fig. 92):

Draw a circle with radius equal to the length of the given side. The radius can be plotted round the circumference exactly six times, and by joining the points so obtained, the figure is obtained.

(By drawing the diagonals, it will be seen that the hexagon contains 6 equilateral triangles.)

To construct a regular hexagon using the tee-square and 60-degree set-square (Fig. 93):

Draw a horizontal line AB equal to the required length of side. From A and B draw lines at 60 degrees to AB to find C, D, E, and F.

To draw a regular octagon within an enclosing square (Fig. 94):

Draw the square $ADCB$ and its diagonals AC and DB. With centres A, D, C and B and radius equal to half the length of a diagonal, draw arcs to cut the sides of the square at E, F, G, H, J, K, L, M. Join F to G, H to J, K to L and M to E. $EFGHJKLM$ is the required octagon.

To construct a regular pentagon with given length of side (Fig. 95):

Draw a semicircle and divide the arc into five equal parts. Draw from the centre A radials through the points of division 1 and 4. Along $A1$ and $A4$ measure required length of sides to find B and C. Bisect BA and AC to find the centre O of a circle which, with radius equal to OA, will enclose the pentagon. By drawing through 2 and 3 from A, the diagonals can be found. (The method can be adapted for the construction of any regular polygon. The semicircle being divided into the same number of equal parts as the sides of the polygon required.)

To construct any regular polygon when the length of the sides, AB, is known (Fig. 96):

Draw AB. With centre B and radius less than AB, describe a semicircle. Divide the semicircle into the same number of equal parts as the required polygon has sides—7 in this example. Draw through point of division 2, BF equal to AB. Bisect AB and BF, and produce the bisectors to meet in O. With centre O and radius OA describe a circle. Draw through points 6, 5, 4 and 3 from B to the circumference to find K, J, H and G, which when joined complete the required heptagon.

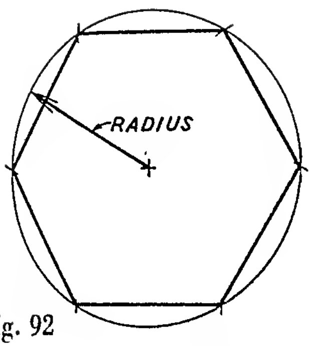


Fig. 92

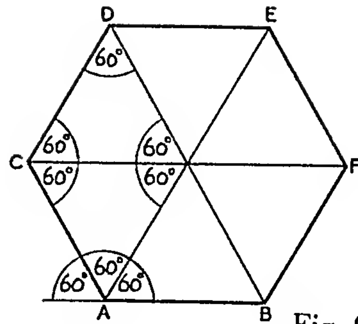


Fig. 93

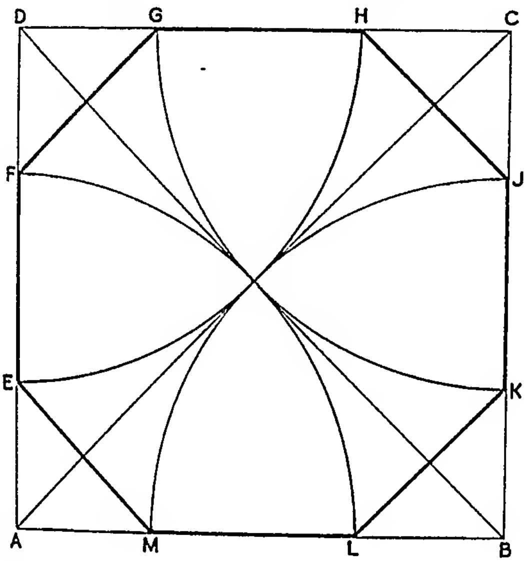


Fig. 94

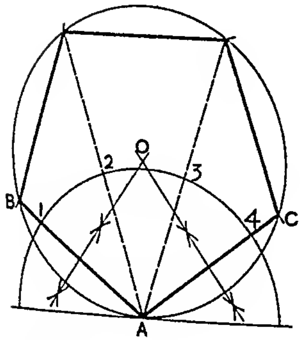


Fig. 95

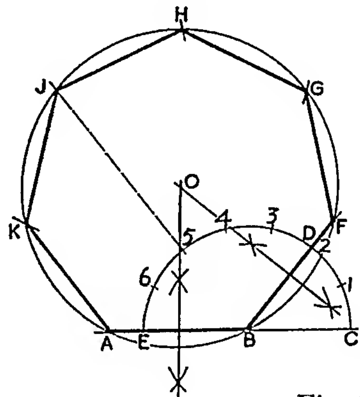


Fig. 96

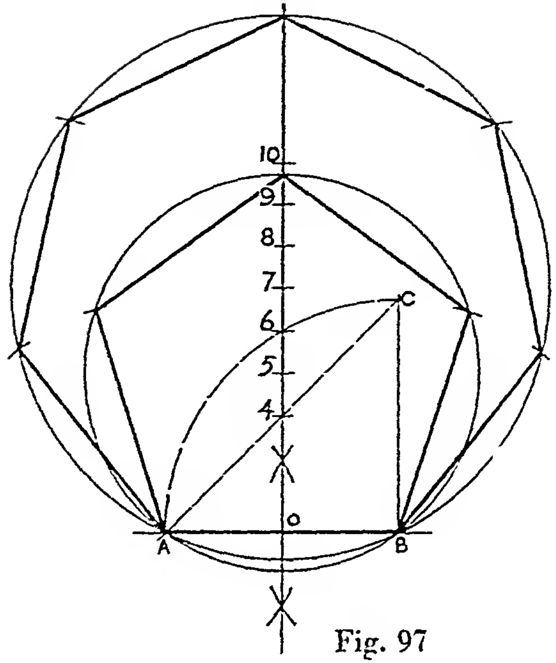


Fig. 97

To construct any regular polygon on a given line, AB, equal to the length of the sides (Fig. 97):

Draw AB. At B draw a perpendicular BC equal to AB. Join C to A. Bisect AB in O, from which point erect a perpendicular of indefinite length, cutting AC in 4. (4 is the centre of a square raised on AB.) With B as centre and radius AB draw a quadrant $A6C$. (6 is the centre of a hexagon raised on AB.) Midway between 4 and 6, point 5 is the centre of a pentagon described on AB, and if points 7, 8, 9, etc., are marked on the perpendicular so that the divisions all equal 4–5 or 5–6, then these points are the centres for the regular polygons of corresponding numbers of sides. (This method is only approximate for regular figures having 5, 7, 8 sides, etc.)

Use of Irregular Polygons in Surveying

Irregular polygons are used chiefly in surveying practice. Field surveying using the chain, plane table and box sextant has been briefly dealt with in previous pages; the use of the theodolite is referred to in the following description.

Fig. 98 shows the method of surveying an irregular field of, say, seven sides. The angles of the field are picketed off with ranging poles at stations A, B, C, D, E, F, G. Firstly, the distances

Fig. 98

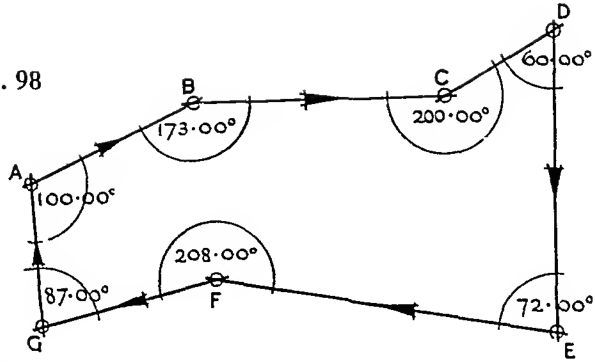


Fig. 99

ANGLE CHECK

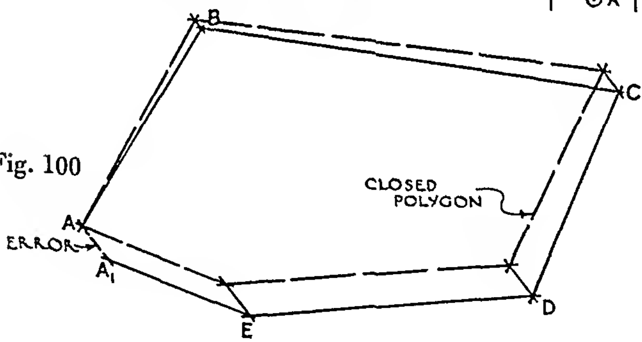
A	100·00
B	173·00
C	200·00
D	60·00
E	72·00
F	208·00
G	87·00
	900·00
+ 4 RIGHT-ANGLES =	360·00
	1260·00 DEGS.

14 RIGHT-ANGLES EQUAL 1260 DEGS.
∴ MEASUREMENTS ARE CORRECT.

FIELD BOOK

⊙A
100°
⊙G
130°
⊙F
220°
⊙E
205°
⊙D
95°
⊙C
150°
⊙B
130°
⊙A

Fig. 100



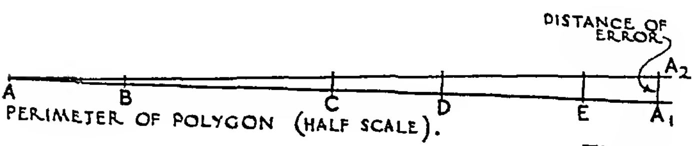


Fig. 101

E

between the various stations are carefully measured and recorded in the field book. Should it be possible, although it rarely is the case, the theodolite is then levelled up in a favourable position in the field so that all stations are visible for taking readings. It is usual, however, to commence at station A, moving round to B, C, D, etc., obtaining the measurement in degrees of the internal angles, known as making a Theodolite Traverse Survey. To obtain a check on the work, it is necessary to traverse round and back to the starting-point, i.e. making a "closed" traverse. The theodolite is normally read using the compass so that the bearing of each line in relation to the points of the compass, i.e. the "magnetic bearing" of each line, is determined.

In Figs. 99–100 the polygonal boundary of the field with the measured internal angles is shown, together with specimen field book entries. The survey can be checked for accuracy of angle measurement by applying the rule respecting angles of any polygon previously mentioned.

Frequently, in the drawing of an irregular polygon as described above, the figure fails to close because of faulty observations, inaccurate measuring or setting out. In Fig. 101 an error A–A_1 is indicated. To adjust this, the perimeter of the polygon is drawn out as a straight line $ABCDEA_1$, as shown—this may have to be done to scale to avoid undue length. At A_1 on the developed perimeter a perpendicular is erected, and made the same distance as the error, A_1A_2. A_2 is joined to A, and perpendiculars cutting A_2A are erected from points B, C, D, E. These perpendiculars show the proportionate amount of error to be allowed for at each angle of the polygon. On the original survey, plotting lines are drawn from each angle parallel to the line of error A–A_1, and by marking off on these parallels the respective proportions of error as determined above, and by joining the points so obtained the polygon is replotted and will now close.

Fig. 102 illustrates an enclosure with its boundary and survey lines. The survey lines have been measured in chains and links, as will be seen from the "field book" extract. See also Chapter V, Fig. 67.

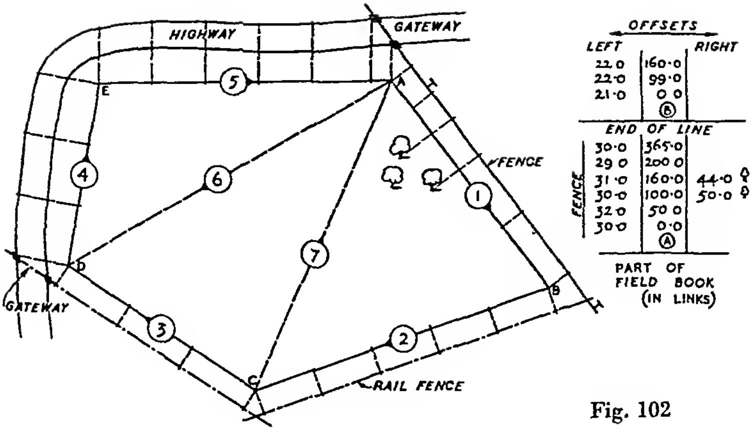

HIGHWAY
GATEWAY
A
B
C
D
E
1
2
3
4
5
6
7
FENCE
GATEWAY
RAIL FENCE
OFFSETS
LEFT
RIGHT
22.0 160.0
22.0 99.0
21.0 0.0
B
END OF LINE
FENCE
30.0 365.0
29.0 200.0
31.0 160.0 44.0
30.0 100.0 50.0
32.0 50.0
30.0 0.0
A
PART OF
FIELD BOOK
(IN LINKS)

Fig. 102

CHAPTER VII

THE CIRCLE

DEFINITIONS OF PARTS; SETTING OUT OF TANGENTS, SEGMENTS, ETC.; CONTINUOUS CURVES; INSCRIBED AND CIRCUMSCRIBED CIRCLES; SETTING OUT OF ARCHES AND OTHER APPLICATIONS

THE circle is a plane figure confined by a curved line known as the *circumference*, all points on which are equidistant from a point known as the *centre* of the circle. Fig. 103 shows circles illustrating the terms defined below:

Arc—A part of the circumference.

Chord—A straight line, shorter than the diameter, terminated by the circumference at both ends.

Diameter—A straight line passing through the centre of the circle and terminated at both ends by the circumference.

Normal—A straight line drawn from any point on the circumference radial to the centre of the circle.

Quadrant—A quarter of a circle in shape and area.

Radius—A straight line drawn from the centre of a circle to the circumference (plural—*radii*).

Sector—A part of a circle contained between two radii which form an angle of less than 180 degrees.

Semicircle—A half circle in shape and area; the part on either side of a diameter.

Segment—A part of a circle contained between a chord and its arc.

Tangent—A straight line touching the circumference of a circle at one point at right angles to a normal at that point.

The Circumference

The circumference of a circle is approximately 3·141 or $3\frac{1}{7}$ times the diameter in length. $3\frac{1}{7}$ is usually expressed by the Greek letter π and the formula for the length of the circumference as $\pi \times D$. For example: the circumference of a 3″ diameter circle is $\pi \times D = 3{\cdot}141 \times 3 = 9{\cdot}423$ inches.

Fig. 104 shows a useful practical method of finding the length of a $\frac{1}{4}$ circumference or smaller segment. A line is drawn from A at 60 degrees to the horizontal to cut the tangent at B produced at C. BC is then the developed $\frac{1}{4}$ circumference. By drawing a perpendicular line from B to meet CA produced a point D is found, from which lines can be drawn through any point on the $\frac{1}{4}$ circumference AB to the line CB to give the development of the arcs.

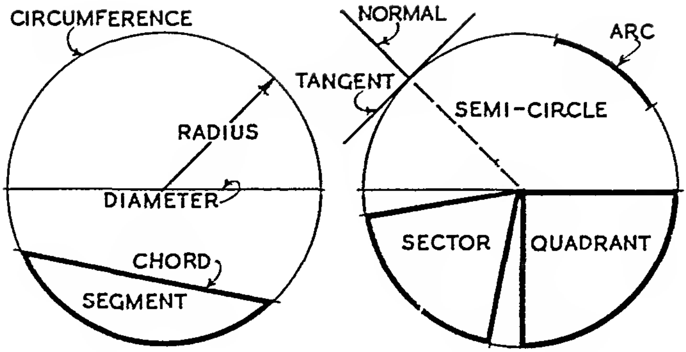


Fig. 103

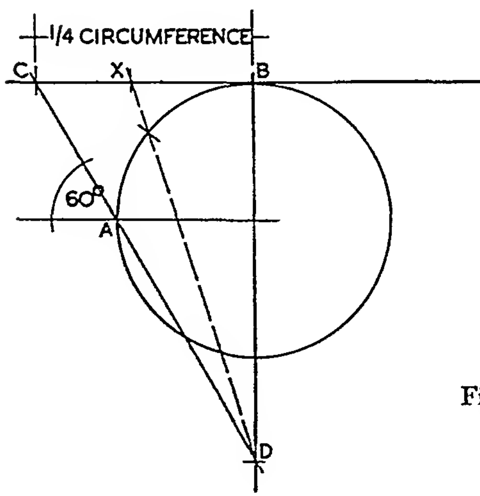


Fig. 104

(Note: Although very slightly inaccurate this method is satisfactory for most practical purposes.)

In geometrical drawing it is common practice to find the development of the circumference of a circle by dividing it into a number of equal units which can be plotted on a straight line. This method is not, of course, accurate, as the plotted units, however small, are chords of the circle.

The Area of a Circle

The area of a circle is found from the mathematical formula— $\pi \times \text{radius}^2$.

For example: the area of a 3″ diameter circle is $3{\cdot}141 \times 1{\cdot}5^2 = 7{\cdot}06$ square inches.

To Find the Radius of a Segment

The mathematical formula for finding the radius of a segment of a circle, the chord and rise being known, is:

$$\text{radius} = \frac{(\frac{1}{2}\text{ chord})^2 + (\text{rise})^2}{2 \text{ rise}}$$

(see Chapter IX, Fig. 164A, for proof.)
A practical application of this arises in the setting out of a segmental wall, the chord of which is, say, 14′ 0″ and the rise, say, 3′ 0″. The radius to be marked off on a batten or lath to be used for describing the curve will be:

$$\frac{(\frac{1}{2}\ 14)^2 + 3^2}{2 \times 3} = \frac{7^2 + 3^2}{6} = \frac{58}{6} = 9'\ 8''.$$

Problems Relating to Circles

✓ *To find the centre of any given circle* (Fig. 105):

Draw AB, a chord of the circle, and bisect it. Produce the bisector to cut the circumference in C and D. CD is a diameter of the circle, and by bisecting it, O, the centre of the circle is found.

To draw a circle to pass through three given points (Fig. 106):

Let A, B and C be the three points. Bisect AB and BC and produce the bisectors to intersect at O, which is the centre of the required circle.

✓ *To draw a tangent to a given circle through a given point A on the circumference* (Fig. 107):

Having determined the centre of the circle, O, draw OA and produce it to B so that AB equals OA. With centres O and B and radius greater than OA, describe arcs to intersect at C. A straight line drawn from C through A is the required tangent.

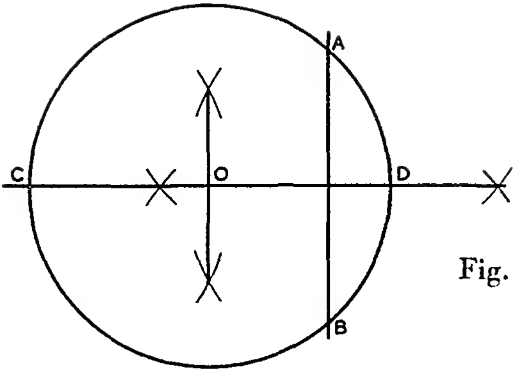


Fig. 105

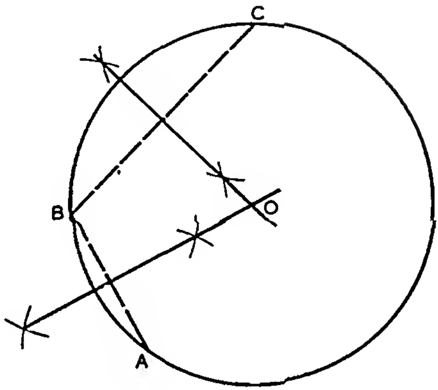


Fig. 106

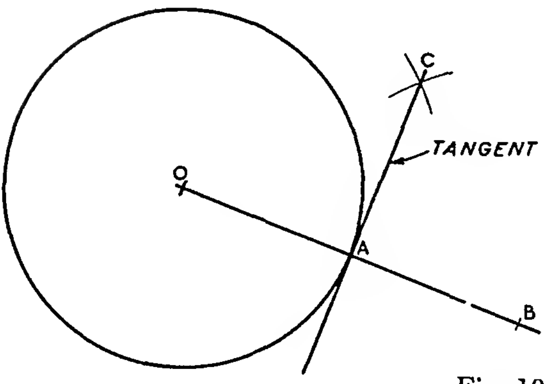


Fig. 107

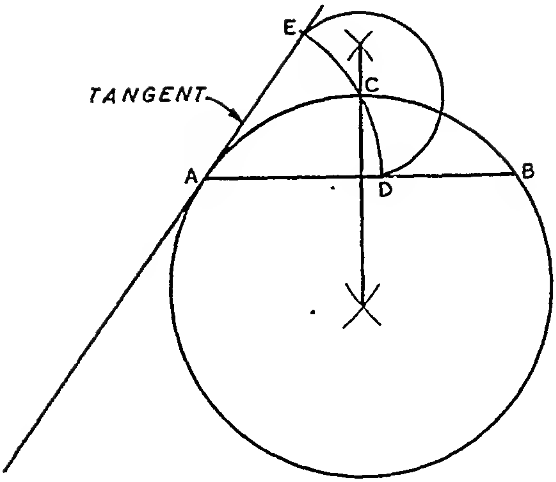


Fig. 108

To draw a tangent to a circle or any segment of a circle through any given point without using the centre of the circle (Fig. 108):

A is the given point on the circumference. Join A to B, any other point on the circumference, and bisect the arc AB to find C. With centre A and radius AC describe an arc to cut AB at D. With centre C and radius CD describe an arc to cut the first arc at E. A straight line drawn from E through A is the required tangent.

To draw a tangent to a circle from a given point A (Fig. 109):

Join A to O, the centre of the circle. Bisect AO in X. With centre X and radius XA describe a semicircle intersecting the circumference of the circle at B. A straight line drawn from B through A is the required tangent.

Alternative method (Fig. 110):

Draw a line through A to cut the circumference of the circle at two points C and B, so that CB is a chord of the circle. With centre A and radius AC describe a semicircle CD. Bisect DB and on it describe a semicircle; from A erect a perpendicular to DB to cut this semicircle at E. With centre A and radius AE describe an arc to cut the circle at F. A straight line drawn from A through F is the required tangent.

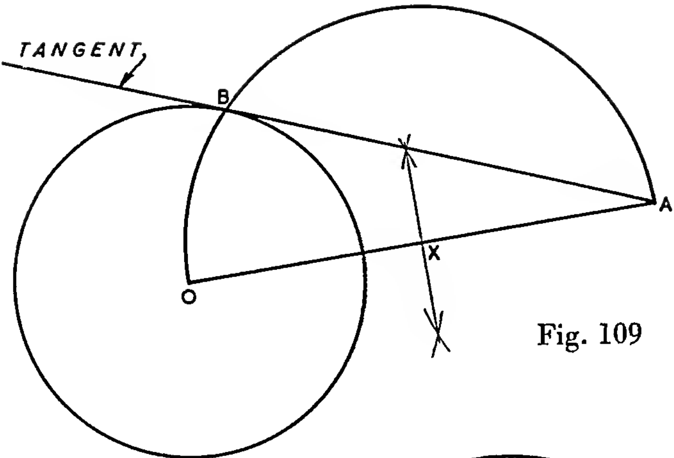
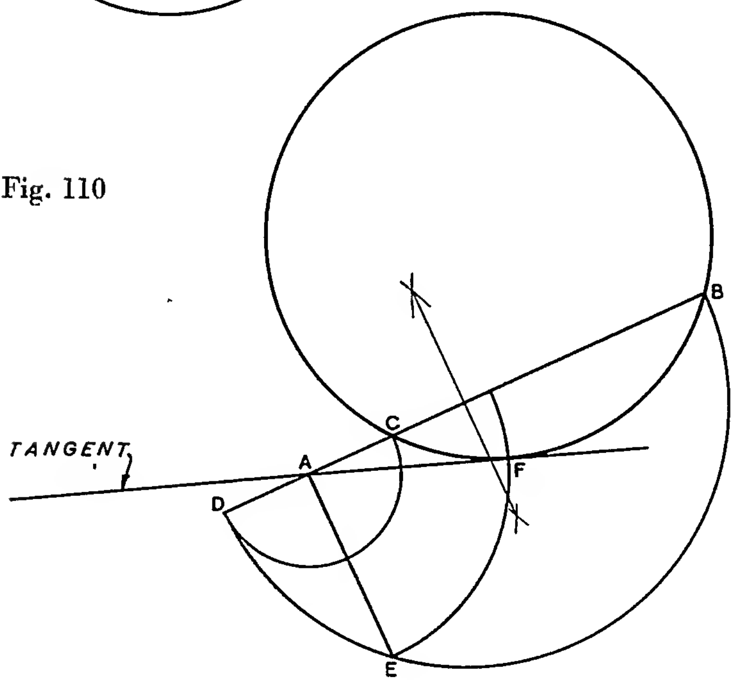


Fig. 109

Fig. 110



To draw a tangent to two equal separated circles (Fig. 111):

Join A and B, the centres of the circles, and bisect AB in C. Bisect AC and CB in D and E respectively. With centres D and E and radius equal to AD, describe arcs to cut the circumferences of the circles at F and G. A straight line drawn through F and G is the required tangent.

To draw a tangent common to two unequal circles in contact with one another (Fig. 112):

Join A and B, the centres of the two circles, and bisect AB in C. With centre C and radius equal to AC, describe a semicircle. Where the circles touch at O erect a perpendicular to AB to cut the semicircle at D. With centre D and radius DO describe a semicircle to cut the circumstances of the two circles at E and F. A straight line drawn through E and F is the required tangent.

To draw a tangent common to two unequal and separated circles (Fig. 113):

Join A and B, the centres of the circles, cutting the circumference of the larger circle in C. Bisect AB in O and with centre O describe a semicircle AB. From C along CA mark point D so that CD equals the radius of the smaller circle. With centre A and radius AD describe a circle to cut the semicircle at E. Draw from A through E to cut the circumference of the larger circle at F. Draw BG parallel to EF. By drawing through F and G the required tangent is obtained.

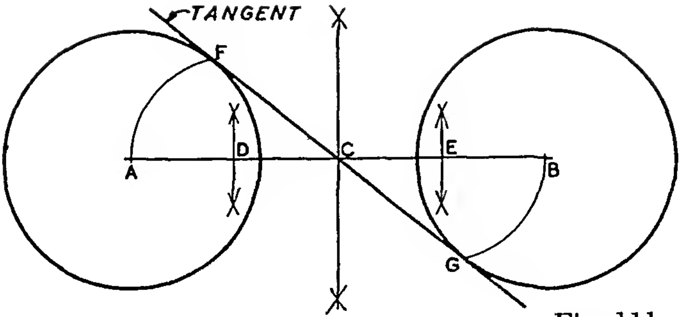


Fig. 111

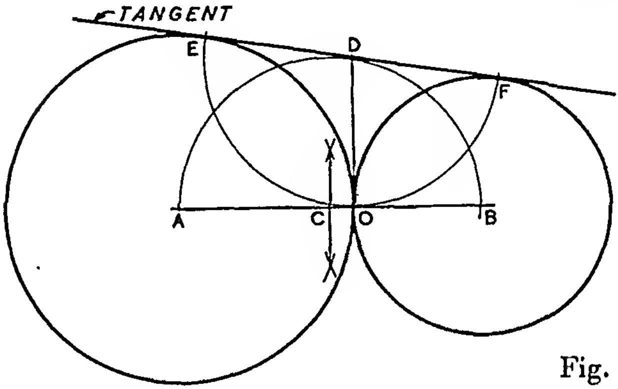


Fig. 112

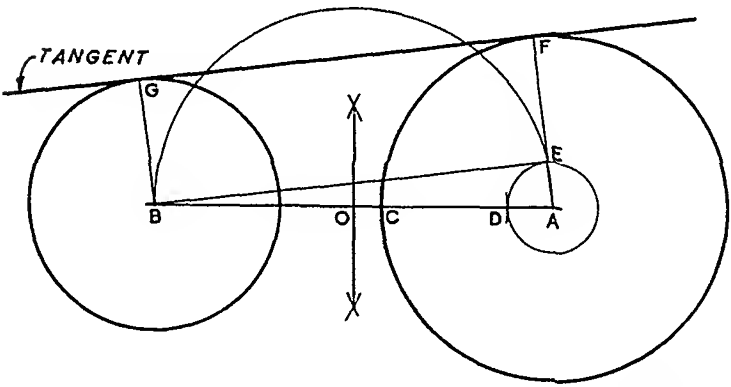


Fig. 113

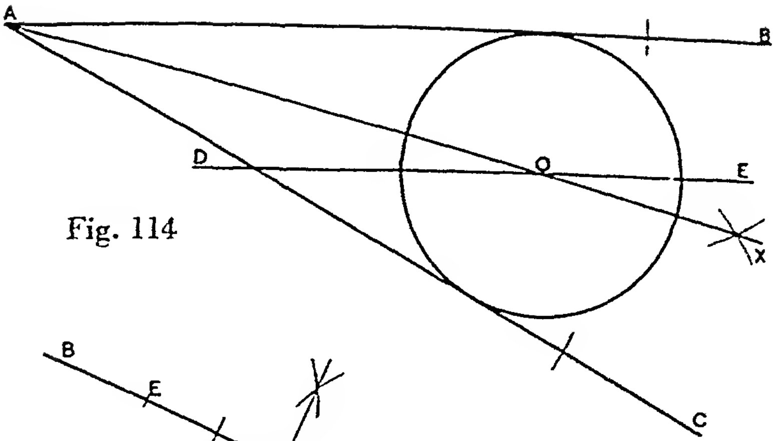


Fig. 114

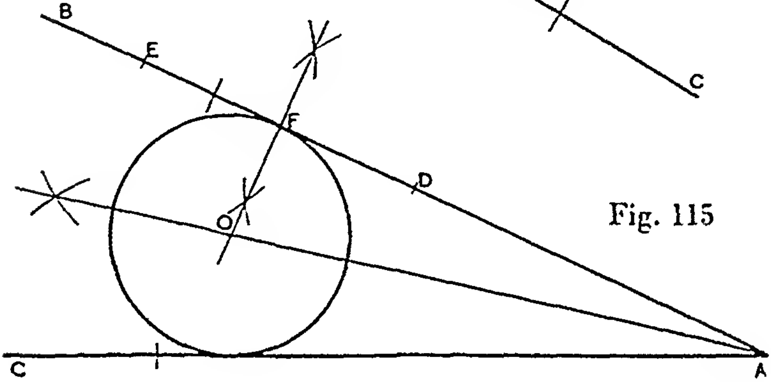


Fig. 115

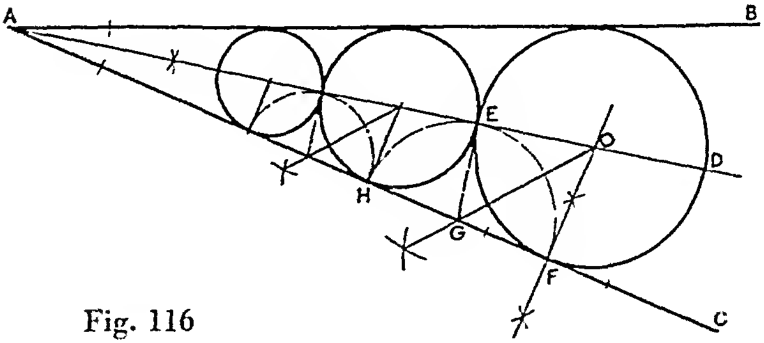


Fig. 116

To inscribe a circle of given radius tangential to two converging lines (Fig. 114):

Draw the lines AB and AC. Bisect the angle CAB. Draw DE parallel to AB, and a distance away from AB equal to the radius of the given circle. Where DE intersects the bisector AX, the centre of the circle, O, is found.

To inscribe a circle of any radius tangential to two converging lines (Fig. 115):

Draw the lines AB and AC. Bisect the angle CAB. Take any two points, D and E along AB. Bisect DE in F, and produce the perpendicular bisector to cut the bisector of the angle at O. Then O is the centre and OF the radius of the circle.

To construct a series of tangential circles within two converging lines (Fig. 116):

The angle formed by the converging lines BAC is bisected and the first circle with centre O is drawn by the method described in the previous example. The angle FOE is bisected, the bisector cutting AC at G. With centre G and radius GF an arc is described to cut AC at H. By drawing from H parallel to FO the centre of the next circle is found along AD, and the method can be repeated for other circles as required.

To inscribe a circle within and tangential to the sides of a triangle (Fig. 117):

Draw the triangle ABC. Bisect the angles ACB and BAC and produce the bisectors to intersect at O, which is the centre for the required circle. Draw the perpendicular from O to CA, to cut CA at D. OD is the required radius of the circle.

Fig. 117

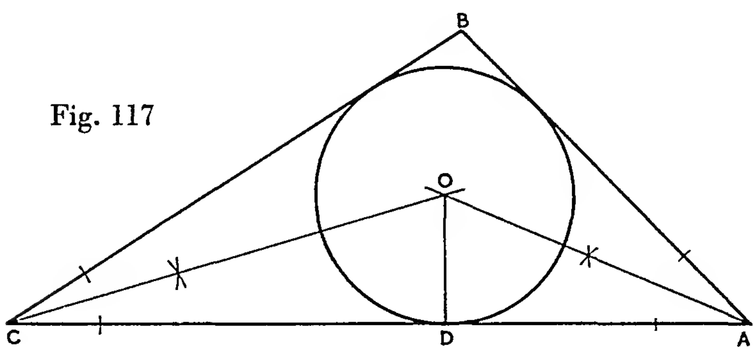


Fig. 118

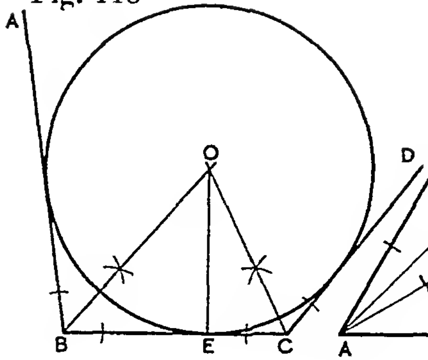
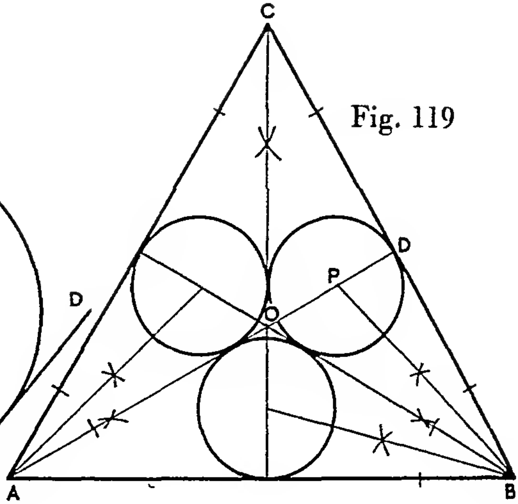


Fig. 119



To inscribe a circle within and tangential to three lines (Fig. 118):
Bisect the angles ABC and BCD to find centre O. Draw the perpendicular from O to BC to cut BC at E. OE is the radius of the required circle.

To inscribe three equal and tangential circles within and tangential to the sides of an equilateral triangle (Fig. 119):
Draw the equilateral triangle ABC. Draw the bisectors of the angles to intersect at O. Continue AO to cut BC at D. Bisect the angle OBC and draw the bisector to cut AD at P. P is the centre and PD the radius of one of the circles. The other circles can be found similarly.

To inscribe six equal and tangential circles within and tangential to the sides of a regular hexagon (Fig. 120):
Draw the hexagon and divide it into six equilateral triangles. Taking one triangle ABO, bisect the angles ABO and AOB to find the centre C and the radius CD of one of the required circles. The others can be found from this.

To inscribe eight equal and tangential circles within and tangential to the sides of a regular octagon (Fig. 121):
Draw the octagon and divide it into eight isosceles triangles. Proceed in a similar manner to the previous example.

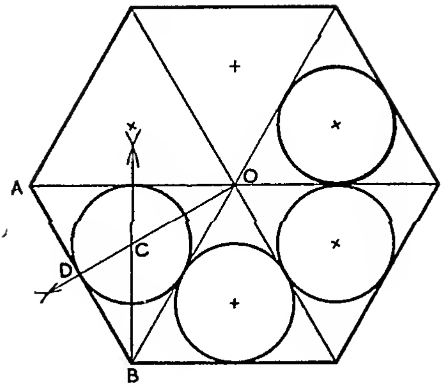


Fig. 120

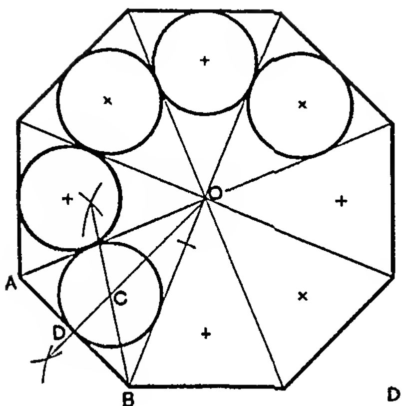


Fig. 121

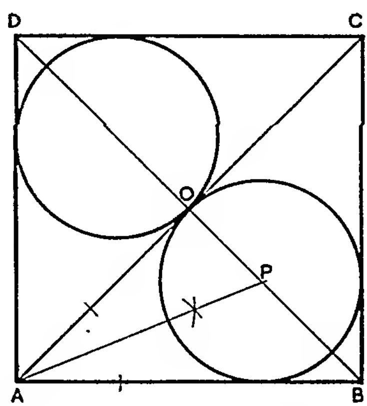


Fig. 122

To inscribe two equal and tangential circles within and tangential to the sides of a square (Fig. 122):

Draw the square $ABCD$, and its diagonals AC and DB intersecting at O. Bisect angle OAB and produce the bisector to cut OB at P. P is the centre and PO the radius of one of the circles; the other can be found similarly.

To inscribe four equal and tangential circles within and tangential to four sides of an octagon (Fig. 123):

Draw the polygon and the diameters, AE, BF, CG, DH. Produce BF to point O. Bisect FOE and produce the bisector to cut AE in P. P is the centre and PE the radius of one of the required circles.

To inscribe three equal semicircles tangential to the sides of an equilateral triangle (Fig. 124):

Draw the triangle ABC. Bisect the sides in D, E and F respectively. Bisect the angle FAB and produce the bisector to cut BD in G. Mark point H on CE and point K on FA so that EH equals FK equals DG, and construct the smaller equilateral triangle GHK on the sides of which the required semicircles can be described.

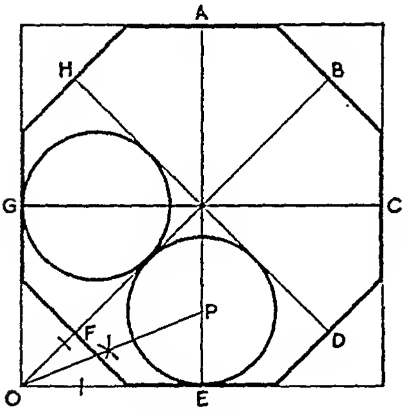


Fig. 123

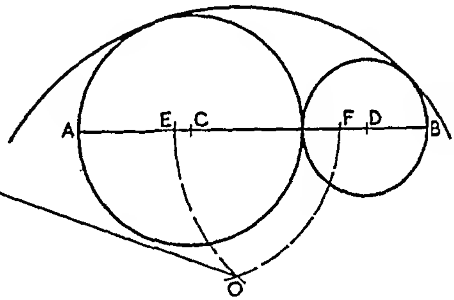


Fig. 125

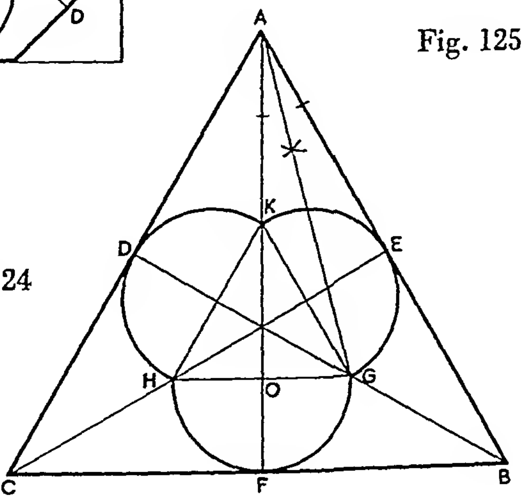


Fig. 124

To draw two circles tangential to each other and to the arc of a greater circle of given radius (Fig. 125):

Draw AB equal in length to the combined diameters of the two circles. Mark off the radii of the circles from A and B respectively to find the centres C and D, and describe the circles which, having a common normal, are tangential. To find the centre of the greater circle mark off its radius OP from A and B, giving AF and BE equal to OP. With centres C and D and radii CF and DE respectively describe arcs intersecting at O, which is the centre of the required circle.

To draw a circle with its centre on a given line AB, to contact a given line CD and a given circle with centre G (Fig. 126):

Draw a line OP perpendicular to CD and mark along it a number of equal divisions (5 in this case). Draw EF a normal to the given circle and from E mark off divisions equal to those along OP. With centre G describe arcs to pass through these divisions and to intersect corresponding co-ordinates projected from OP parallel to CD. Through the points of intersection draw a smooth curved line; this line is the locus of centres of circles tangential to CD and the given circle, and where it cuts AB at H is the actual centre of the circle required. (Definition of a locus—if a point moves so as to satisfy some geometrical condition the line which it describes is called the locus of the point.)

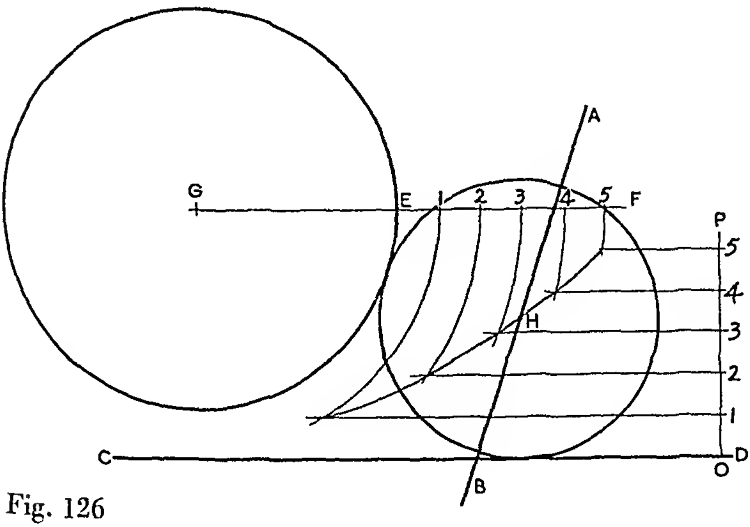
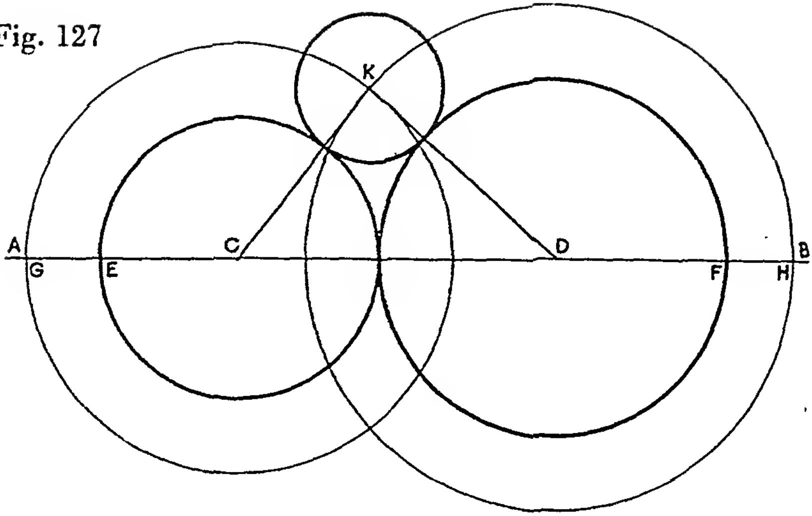


Fig. 126

Fig. 127



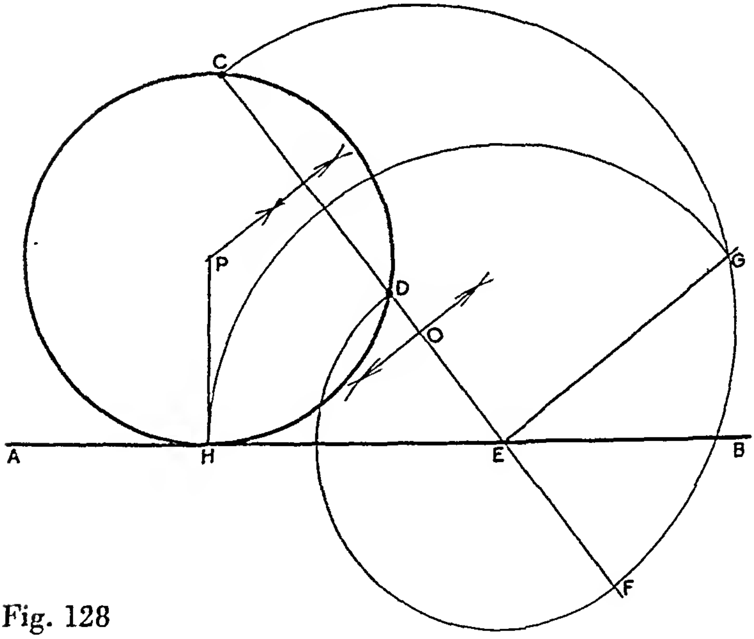


Fig. 128

To draw three unequal circles tangential to one another externally (Fig. 127):

Draw a straight line AB and mark off CD equal to the combined radii of the two larger circles. Describe the circles with centres C and D respectively cutting AB at E and F. From E and F mark off distances EG and FH equal to the radius of the third circle, and with centres C and D describe circles of radius CG and DH respectively, to intersect at K. K is the required centre of the third circle.

To draw a circle to pass through two given points, C and D, and to contact a given line AB—which is not parallel to a line joining C and D (Fig. 128):

Draw a straight line from C through D to E on AB, and continue to F, so that EF equals DE. Erect a perpendicular to DF at E. Bisect CF in O. With centre O and radius OC describe a semicircle to cut the perpendicular in G. With centre E and radius EG describe an arc to cut AB at H. Erect a perpendicular to AB at H, to intersect the bisector of CD at P. P is the centre and PH the radius of the required circle.

To draw any number of continuous arcs to pass through any number of given points (Fig. 129):

The centre A of the first arc which passes through points 1, 2 and 3, is found by the method described on p. 62, Fig. 105. The centre B of the second arc is found by bisecting 3, 4, so as to intersect a normal to the first arc from 3, and so on. The centres of adjoining arcs must have a common normal.

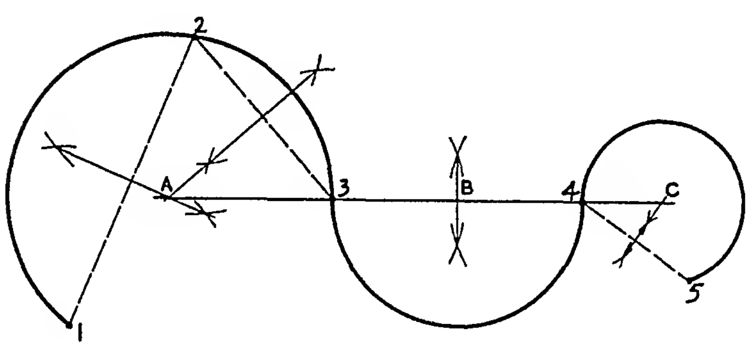


Fig. 129

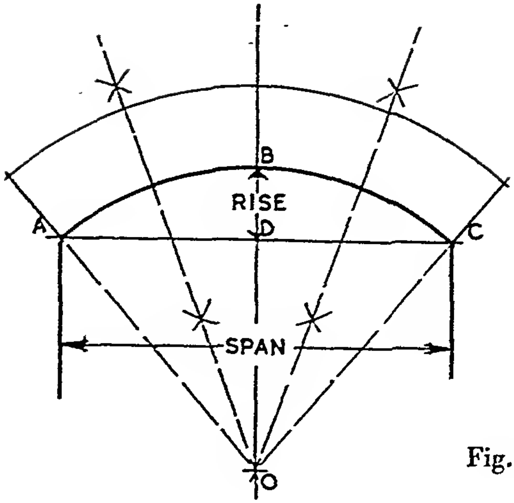


Fig. 130

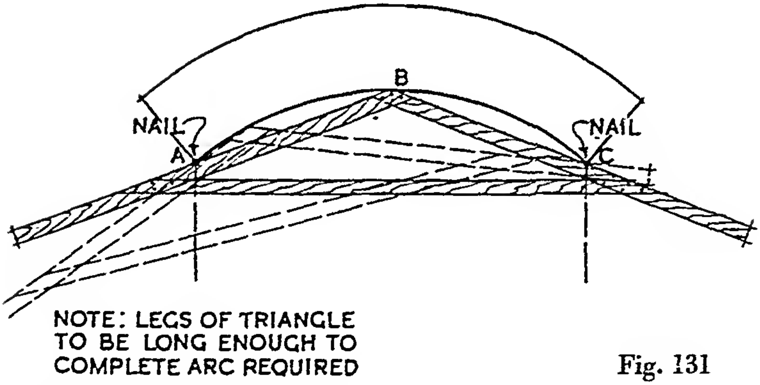


Fig. 131

Practical Uses of the Circle

There are various methods of finding the shapes of turning pieces, shaped lintels, wood centres, etc., for segmental arches. Fig. 130 illustrates the method of finding the curve of the soffit of a segmental arch, the span of which is AC and the rise DB. With compasses AB and BC are bisected. The intersection of the bisecting lines, O, is the centre for striking the required arc. Although this method can be used in practice it is more suitable for drawing plans, etc.

A more practical method is shown in Fig. 131. Taking an arch of similar proportions to the foregoing pieces of thin batten are

formed into a triangle of the form of the triangle ABC, as illustrated—or, if the arch is flat enough, a single wide board can be so shaped. Nails are fixed at points A and C on the timber to be cut, and with a pencil held at the apex B of the triangle the required curve can be drawn by moving the triangle, keeping the sides against the nails, as shown.

Fig. 132 illustrates the method of drawing the outline of an ogee (double curve) dome or arch. The span and height being determined, a line AB is drawn from the springing to the apex. A point C is marked along this line where it is desired the curve should change direction, and AC and CB are bisected. The intersections of the bisectors with horizontal lines drawn through A and B, as shown, give the centres (1 and 2) for the required arcs for one side of the dome or arch. The centres for drawing the other side can be plotted as shown by the broken lines.

Fig. 133 shows the setting out of the outline for a W.C. seat. AB, the width, is bisected at C and a circle with centre C and

Fig. 133

Fig.

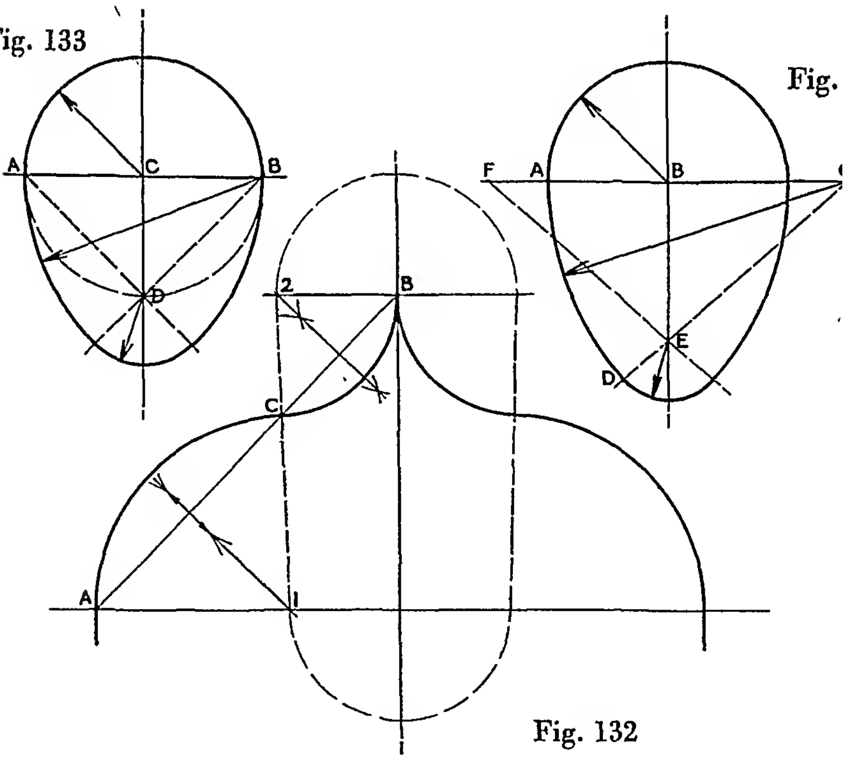


Fig. 132

radius equal to AC is described. Lines are then drawn from A and B through D, the point where the circle is cut by the perpendicular bisector of AB. A, B and D are then used as centres for arcs to complete the figure as shown.

Fig. 134 shows the setting out of the outline of an egg-shaped sewer. The method is similar to that of the foregoing. FA is equal to half AB, and FE is equal to twice AB.

Fig. 135 shows the setting out of the section of a handrail. An equilateral triangle ABC, is constructed with sides equal to the width of the handrail, and a rectangle, $ABED$, is constructed on AB which is then divided into 5 equal parts. Lines are drawn from one division from A and B to D and E respectively. Where these lines, $1D$ and $4E$, intersect AC and BC, centres are found at F and G for the construction of arcs as continuations of an arc described with centre C. The section is then completed by further arcs of radius equal to those struck from F and G but with centres at H and K, and straight lines as shown.

Fig. 136 is a drawing of a typical standard steel beam section—10″ ×5″ Rolled Steel Joist—showing the use of arcs in its setting out.

Fig. 137 shows how herring-bone strutting can be easily drawn and the cuts determined. At the intersection of the diagonals drawn from the top and bottom of the joists describe a 2″ diameter circle, and draw tangent lines to this circle from points on the sides of the joists ½″ away from the floor boards and ceiling.

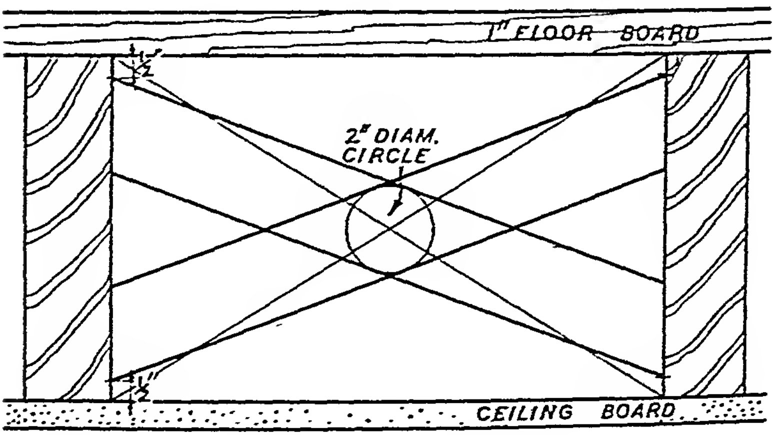


Fig. 137

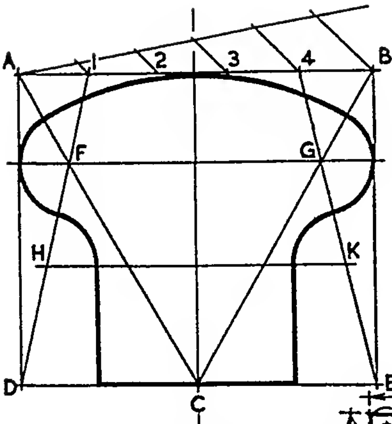


Fig. 135

HANDRAIL SECTION

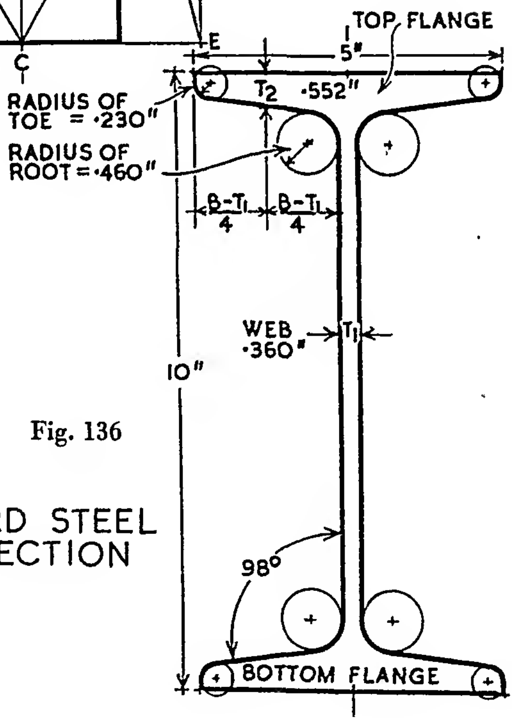


Fig. 136

STANDARD STEEL BEAM SECTION

CHAPTER VIII

THE ELLIPSE

METHODS OF SETTING-OUT TRUE AND PSEUDO ELLIPSES: SETTING-OUT OF 3- AND 5-CENTRED ARCHES AND OTHER PRACTICAL APPLICATIONS

The Ellipse

Considered as a plane curve, the ellipse is the locus of a point tracing a continuous curve in such a way that the sum of the distances from the point to two fixed points or foci is always constant.

When a cone or cylinder is cut by an inclined plane the outline of the section obtained is an ellipse—as shown in the photograph, Plate 4, Chapter XIX.

The curve of the ellipse in whole or part is considerably used in building design. and a knowledge of its setting out and its properties is essential to the draughtsman and the practical man.

There are many methods of setting out the ellipse; the more important for various purposes are given below.

Trammel Method of Setting-out Ellipses

This is suitable for almost all purposes. Fig. 138 illustrates the application to drawing. The lengths of the major axis AB and the minor axis CD having been determined and drawn at right-angles to one another so that they intersect at O midway along each axis, a straight strip of stout paper (the trammel) is taken and a distance equal to half the major axis is marked on it, and within this distance half the minor axis, CO. is also marked off from one end. By placing the strip of paper on the drawing so that the two marks separated by the difference between the distances fall on the axes as shown, and then by moving the paper. but always keeping the marks on the axes, the third mark can be made to trace the curve of the ellipse.

Fig. 139 shows how the method can be applied for marking out an ellipse on, for example, a plot of land, using grooved planks in the directions of the major and minor axes, and with a batten as trammel arranged to slide in them.

Other Methods of Setting-out Ellipses

Fig. 140 shows the setting-out by means of two circles with a common centre O and diameters equal to the major and minor axes. Draw any convenient number of radials cutting both circles and from the points so obtained draw lines parallel to the axes. The intersections of these lines give points on the curve of the ellipse.

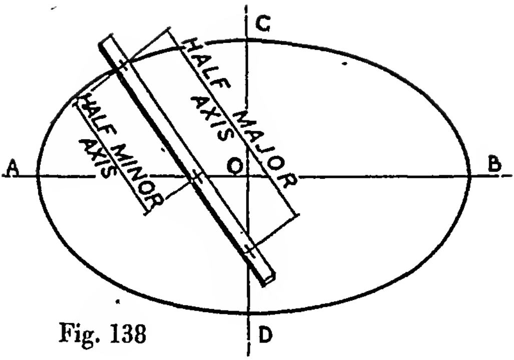


Fig. 138

SETTING OUT OF ELLIPSE USING TRAMMEL

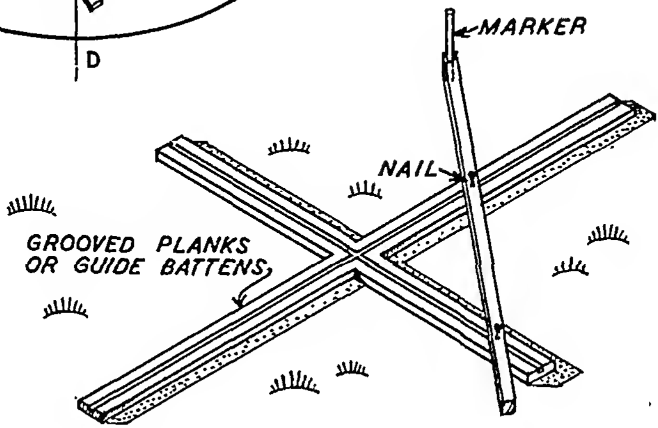


Fig. 139

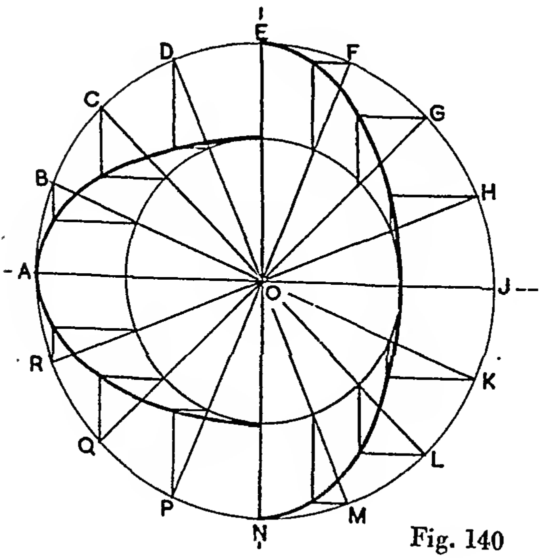


Fig. 140

Fig. 141 illustrates a similar method. The axes, AB and CD, and the foci, F and F^1, are given. The distance between the foci is divided into an odd number of units—in the example seven are shown—which are indexed. Then the procedure is to take compasses and with radius equal to $A1$ and with F^1 as centre to describe an arc, and with radius equal to $B1$ and centre F to describe another arc to cut the first one. Similarly with radii equal to $A2$ and $B2$, and so on as shown. The intersections of the arcs give points on the curve which can then be drawn.

In Fig. 142 the major and minor axes, AB and CD, having been drawn, the foci are found by using compasses and with radius equal to AO and with centre C by cutting the major axis at F and F^1. Assuming the ellipse is being drawn on a piece of plywood, then nails are fixed at F, F^1 and C and a length of string is fastened to F and F^1 and passed tightly round C. By releasing the nail at C and putting a pencil in the loop the curve can be drawn by moving the pencil around the foci, keeping it tightly against the string.

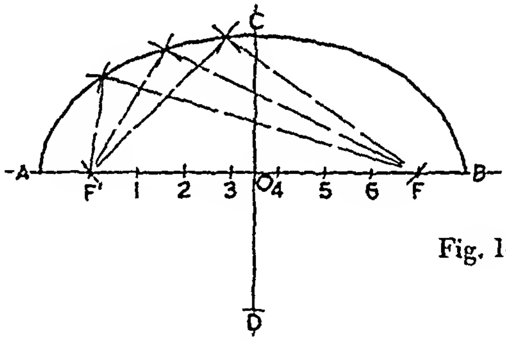


Fig. 141

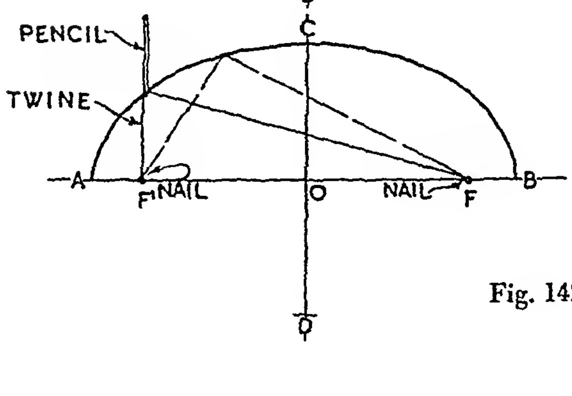
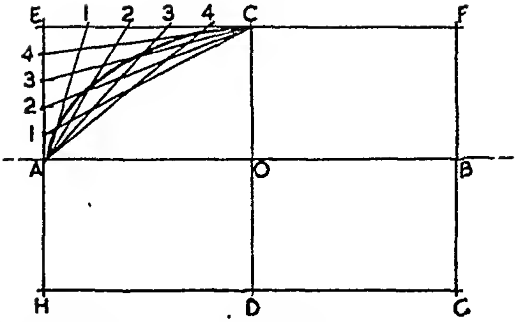


Fig. 142



SETTING OUT OF ELLIPSE

Fig. 143

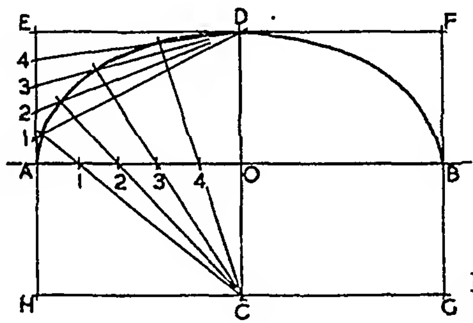


Fig. 144

Although somewhat inaccurate, Fig. 143 shows the construction of an ellipse within a rectangle. The rectangle *EFGH* is drawn about the major and minor axes, *AB* and *CD*, as shown. *AE* and *EC* are both divided into the same number of equal parts, which are indexed. Then by drawing from *A* to the division points marked on *EC* and from *C* to the division points marked on *AE* intersections are made through which a quarter of the curve of the ellipse can be drawn. The procedure is repeated in the remaining parts of the rectangle.

Fig. 144 illustrates an alternative method of drawing an ellipse in a rectangle. The method is somewhat similar to the foregoing and can be understood from the diagram. The segment of an ellipse can also be drawn in the same way as shown in Fig. 145, in which *DC* represents the major or minor axis and *AEFB* the rectangle in which the segment is to be drawn. The same principle can be again employed in setting out a rampant or raking ellipse, as shown in Fig. 146, or alternatively, the method illustrated in Fig. 143 can be employed.

SETTING OUT OF ELLIPSE

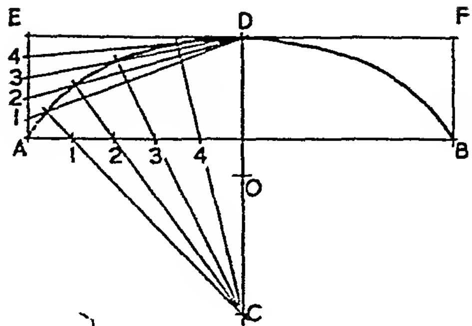


Fig. 145

RAKING ELLIPSE

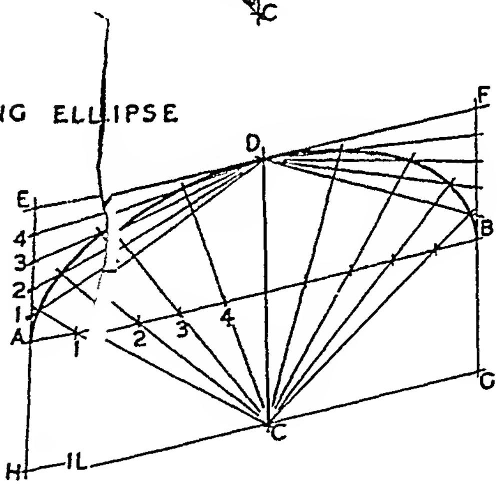


Fig. 146

To determine the axes of a given ellipse (Fig. 148):

Draw any two parallel and equal chords AB and CD to the ellipse. Bisect these chords and draw the bisector EF. Bisect EF in O. With centre O and any reasonable radius describe a circle to cut the ellipse in G, H and K. Parallel lines to GH and HK through O are the required axes.

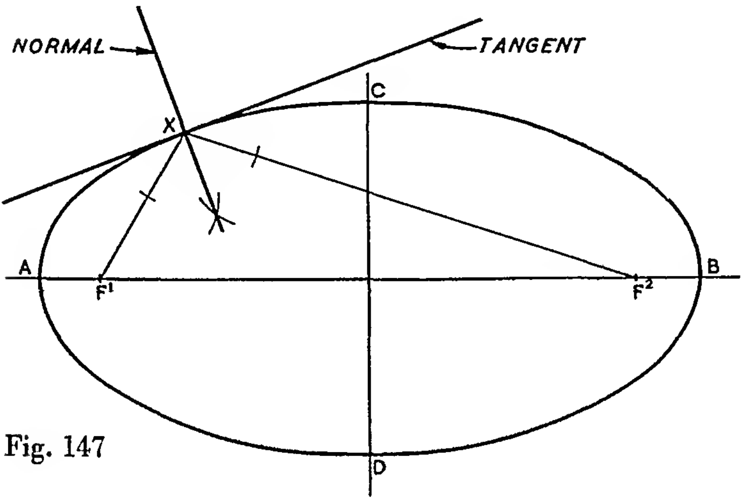


Fig. 147

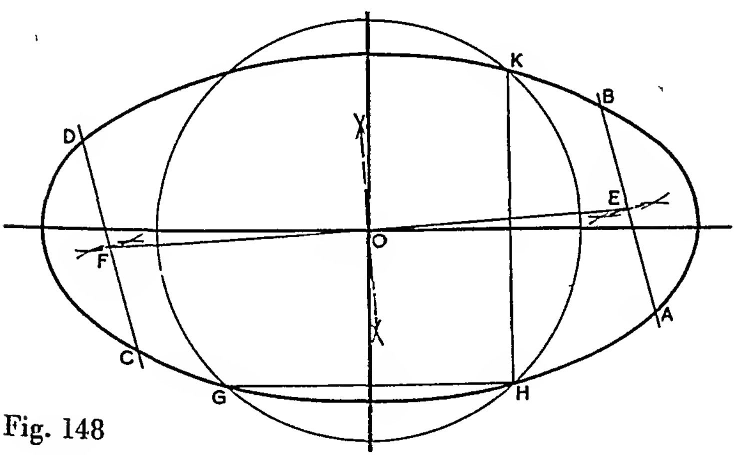


Fig. 148

Pseudo-Elliptical Arches

Arches of true semi-elliptical curve are seldom used in masonry and brick construction owing to the expense of cutting the stones or gauged bricks, and curves made up of arcs approximating to a semi-ellipse are used. Two types are described below.

3-Centred Arch

Fig. 149 illustrates a 3-centred elliptical arch in masonry. *AB* is the clear span which is divided into four equal parts to find centres 1 and 2. From these centres tangent circles are described. From each circle tangent lines at 45 degrees to the horizontal are drawn to intersect and so give centre 3. With centre 3 and radius extended to become tangent to the circles an arc can be drawn to complete the arch. Note the radiating of the joint lines from centres 1, 2 and 3.

5-Centred Arch

This more nearly approximates to a true semi-ellipse. In Fig. 150 the method is shown applied to the formation of a gauged brick arch, there being three sets of bricks for the five segments of the arch. *AB* is the clear span of the arch and *OC* is the rise. *AO* is divided into three equal parts, and the divisions are indexed 1 and 2. At *A* a perpendicular is drawn equal to *CO* and is also divided into three equal parts, the divisions being indexed 1^1 and 2^1. From a point along the vertical axis at a distance from *O* equal to *OC*, lines are drawn through 1 and 2 to contact corresponding lines drawn from *C* to 1^1 and 2^1, giving points *D* and *E*. *DC* is bisected and the bisector is produced to cut the vertical axis at *F*. *F* is joined to *D*. *DE* is bisected and the bisector is produced to cut *DF* at *G*. *G* is joined to *E*, cutting *AO* at *H*. Points *K* and *L* are plotted on the other side of the vertical axis to correspond to *H* and *G*. *F*, *H*, *G*, *K*, *L* are then the five centres for drawing the arcs to complete the curve.

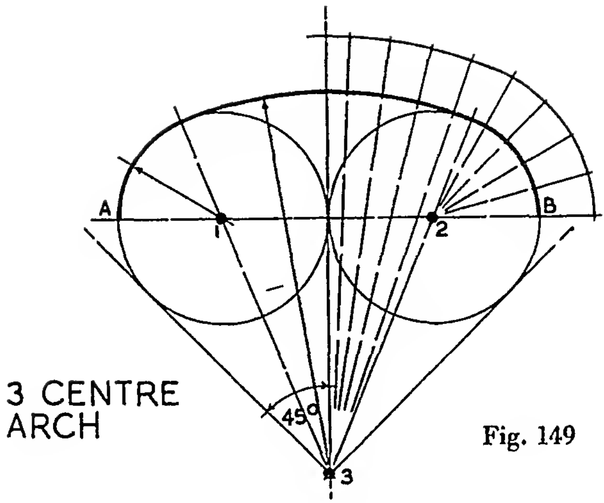


Fig. 149

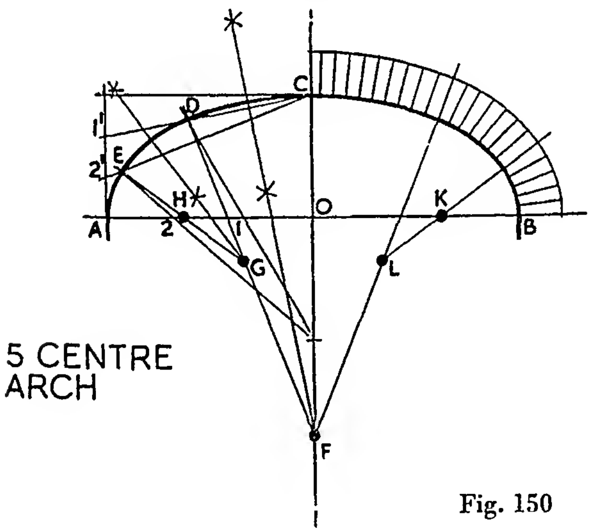


Fig. 150

CHAPTER IX

AREAS

Determining of Areas of Various Plane Figures; Subdivision of Areas

The area of any plane figure is the sum space enclosed by its boundaries. The length of the boundaries is known as the *perimeter*, except in the case of a circle, when it is known as the *circumference*.

Areas are measured in units of square measure, e.g. square metres, square inches, square feet, square yards, square chains, square miles and acres.

The following are the methods of calculating the areas of regular plane figures:

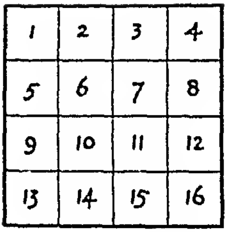
The Square. The length of one side (all sides are equal) is multiplied by itself. In Fig. 151 if the square is assumed to have sides 4" long, the area is 4" × 4", or 16 square inches.

The Rectangle. The length is multiplied by the breadth. In Fig. 152, assuming the rectangle is 10" long and 4" in breadth, the area is 10" × 4", or 40 square inches.

The Parallelogram. In Fig. 153, *ABCD* is a rhomboid. If perpendiculars *AE* and *BF* are drawn, it can be seen that the area of triangle *ADE* equals that of triangle *BCF*, therefore the area of the rectangle *ABFE* equals that of the parallelogram *ABCD*, and the formula for calculating the area of any parallelogram is base multiplied by vertical height.

The Triangle. The formula for calculating the area of any triangle is: base multiplied by vertical height divided by two. This is illustrated by Fig. 154. *ABC* is the triangle. If another triangle, *ACD*, of the same size and having a common side is drawn as shown, it will be seen that a parallelogram *ADCB*, with twice the area of *ABC*, is obtained. From Fig. 153 we know that the area of any parallelogram is given by base multiplied by vertical height, therefore the area of the triangle must be base multiplied by vertical height divided by two.

The Circle. The area of a circle can be determined either graphically or mathematically. Graphically, one method shown



SQUARE

Fig. 151

RECTANGLE

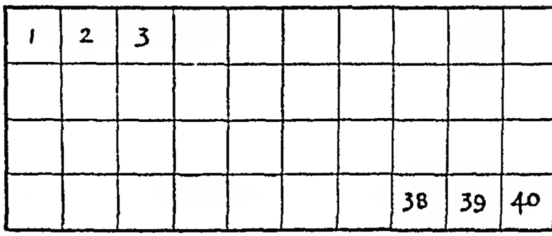
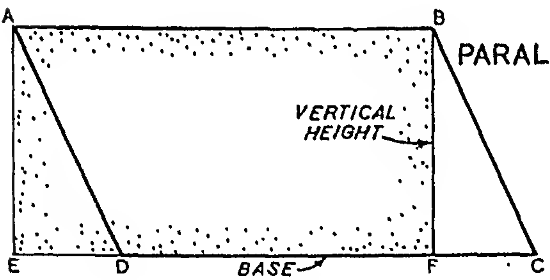


Fig. 152



PARALLELOGRAM

Fig. 153

TRIANGLE

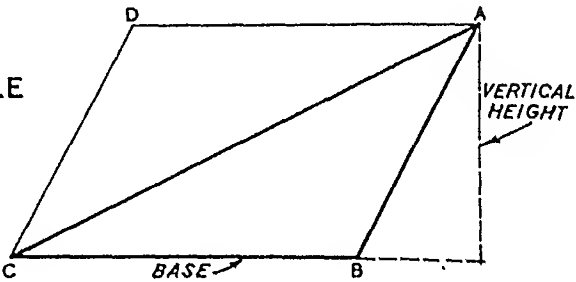


Fig. 154

in Fig. 155 is to divide the circle into a number of equal sectors—24 in this case—and to draw a rectangle with one side equal to the radius and the other to 12 times the chord of the sectors. It will be seen that the 24 sectors, except for the difference between the chords and the arcs, are contained within this rectangle, the area of which approximates to that of the circle.

Mathematically, the area of a circle can be calculated from the formula: $\pi \times \text{radius}^2$, or 3·1416, multiplied by the length of the radius multiplied by itself. This is illustrated graphically by Fig. 156.

To obtain the area of the annulus of a circle, e.g. in Fig. 157, the shaded area represents a 20′ 0″ diameter and 3′ 0″ wide concrete path, the area of which is found as follows:

$$
\begin{array}{lll}
(a)\ \text{Area of outer circle} & =(3{\cdot}1416 \times 10^2) & \text{sq. ft.} \\
 & =(3{\cdot}1416 \times 1\ 0) & ,, \\
 & =314{\cdot}1600 & ,, \\
(b)\ \text{Area of inner circle} & =(3{\cdot}1416 \times 7^2) & ,, \\
 & =(3{\cdot}1416 \times 49) & ,, \\
 & =153{\cdot}9384 & ,, \\
\text{Subtract } (b) \text{ from } (a) & 314{\cdot}1600 & ,, \\
 & \underline{153{\cdot}9384} & ,, \\
\therefore \text{Area of Annulus} & =\underline{160{\cdot}2216} & ,,
\end{array}
$$

To obtain the area of a sector, *ABCD*, of a circle, Fig. 158. Measure the angle *CDA* and divide by the number of degrees in the circle (360 degrees) and multiply the result by the area of the circle. In the example, a simple one, the calculation is $\frac{60}{360}$ degrees $= \frac{1}{6}$ of the area of the circle is the area of the sector.

To obtain the area of a segment, *ADB*, of a circle, Fig. 159, find the area of the sector *ADBC* and then subtract the area of the triangle *ABC*.

The Ellipse. The area of an ellipse is found in a similar manner to that of the circle. The formula is 3·1416 multiplied by half the major axis multiplied by half the minor axis.

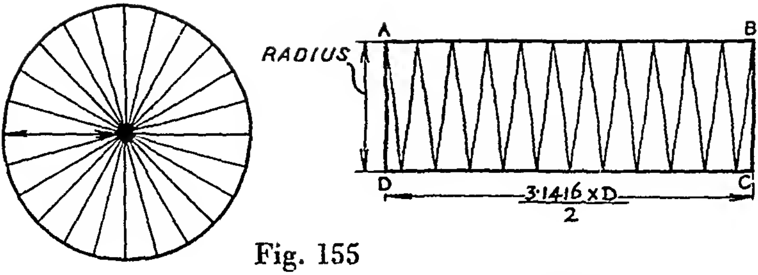


Fig. 155

CIRCLE

AREA = $3\frac{1}{7}$ X RADIUS^2

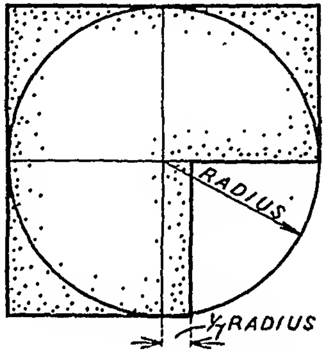


Fig. 156

Fig. 157

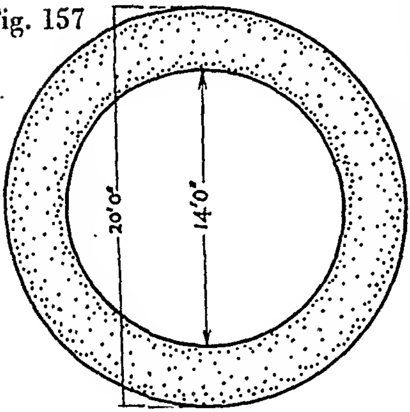
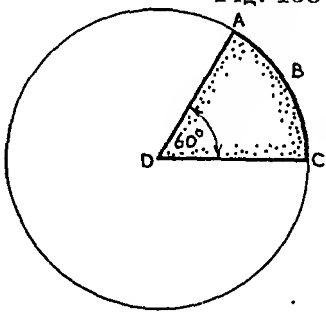


Fig. 158



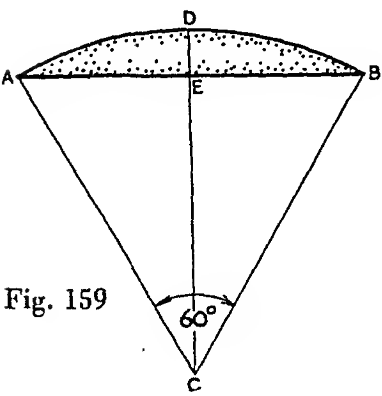


Fig. 159

Miscellaneous Exercises

To construct a triangle equal in area to a given triangle but of greater or less altitude (Fig. 160):

ABC is the given triangle and EF the altitude of the required triangle. Join F to B and from A draw a line parallel to FB to cut CB produced at D. Join F to D. CFD is the required triangle.

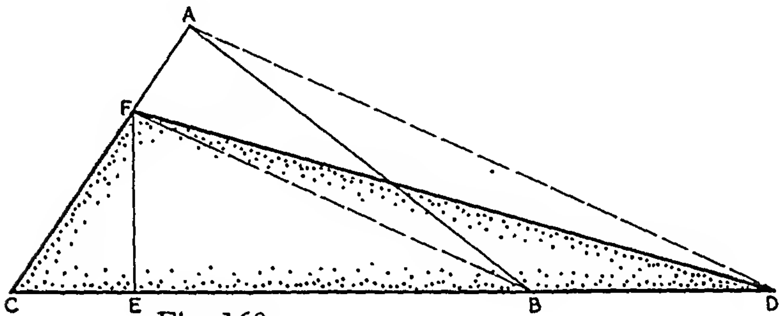


Fig. 160

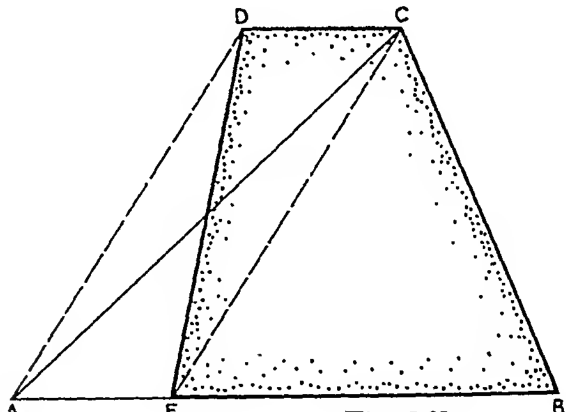


Fig. 161

To construct a trapezium equal in area to that of a given triangle (Fig. 161):

ABC is the given triangle and DC the top of the trapezium. Join D to A and draw a line from C parallel to DA to cut AB at E. $DCBE$ is the required figure.

To construct a triangle of area equal to that of a given regular pentagon (Fig. 162):

ABCDE is the pentagon. Join *AC* and *AD*. From *E* draw a line parallel to *AD* to cut *CD* produced at *G* and from *B* draw a line parallel to *AC* to cut *DC* produced at *F*. Join *A* to *G* and *F*. *AFG* is the required triangle.

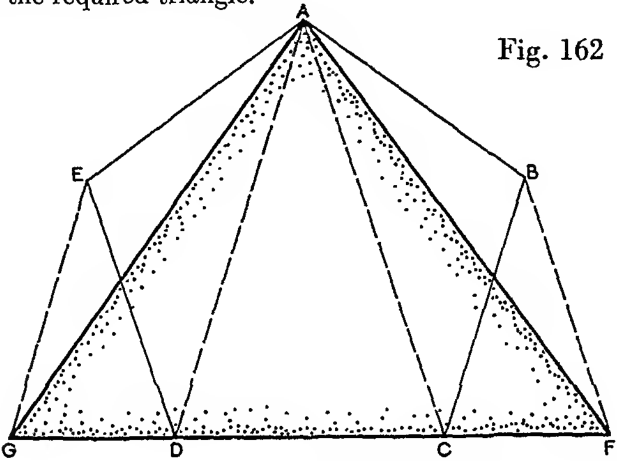


Fig. 162

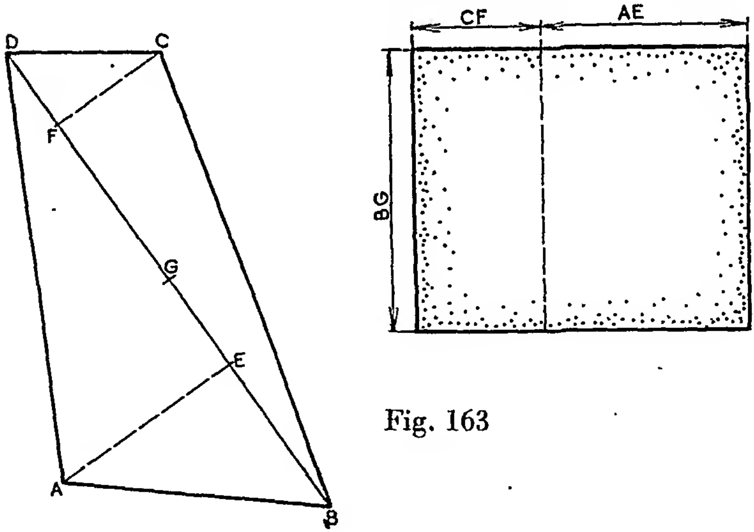


Fig. 163

To construct a rectangle equal in area to a given irregular quadrilateral (Fig. 163):

ABCD is the quadrilateral. Join *B* to *D*. From *C* and *A* draw lines perpendicular to *BD* to cut *BD* at *F* and *E*. Bisect *BD* in *G*. To draw the required rectangle make the base line equal $AE+CF$ and the height equal *BG*.

To construct a triangle equal in area to a given circle (Fig. 164): Draw AB, the base line, equal to the circumference of the circle (by calculation or by plotting). Bisect AB in D, and draw the perpendicular DC equal in length to the radius. Join C to A and B. ABC is the required triangle.

To construct a square equal in area to a given circle (Fig. 165): Draw AB equal to half the circumference of the given circle. Produce AB to C making BC equal to the radius of the circle. Bisect AC and describe a semicircle. Erect a perpendicular from B to cut the semicircle. This line is the length of the side of the required square.

To construct a circle equal in area to the areas of two given circles (Fig. 166):
Construct the right-angled triangle ABC so that AB equals the diameter of one given circle and BC equals the diameter of the other given circle. Describe these circles with centres D and E. Bisect AC in F. The required circle is constructed with F as centre and FA as radius.

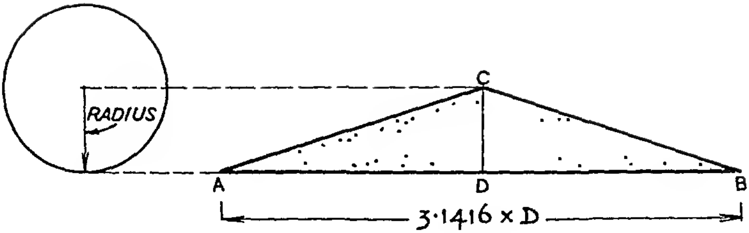


Fig. 164

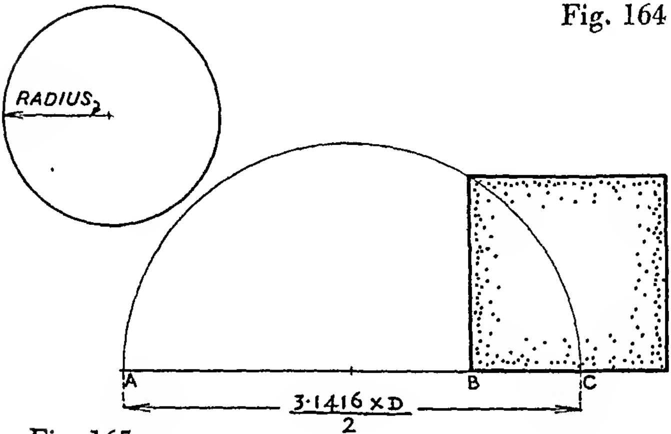


Fig. 165

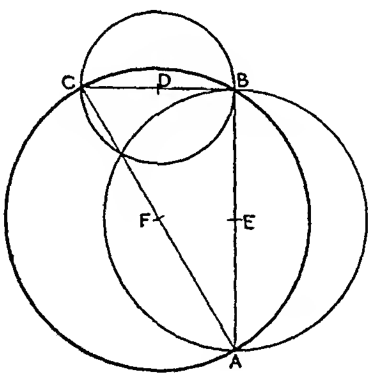


Fig. 166

CHAPTER X

SPECIAL CURVES

PARABOLA; HYPERBOLA; SPIRAL CURVES: ARCHIMEDEAN SPIRAL; SPIRAL BASED ON QUADRANTS: HELIX: ENTASIS OF COLUMNS

THE PARABOLA

THIS curve is the outline of the section obtained when a cone is cut parallel to its inclination (Fig. 167). It is the curve which a suspended cable assumes; the curve of a bending moment diagram for an evenly distributed load in structural calculations; and is the most static arch form. The curve is widely used in design because of its properties in reflecting sound, light and heat.

Setting Out the Parabola

Fig. 168A shows a method of drawing a parabola. *AB* is the width or base line and *CD* the height of the curve. On *AB* a rectangle *AEFB* is constructed, the height being equal to *CD*. *AE* is divided into a number of equal parts and *AC* is divided into the same number of parts. Lines are drawn from the points of division on *AE* to *D*, and from the points on *AC* perpendicular are erected. Where intersections are made points, as shown, through which one half of the curve can be drawn are found. The procedure is completed to complete the whole curve.

Fig. 168B shows the method of drawing a parabola when the

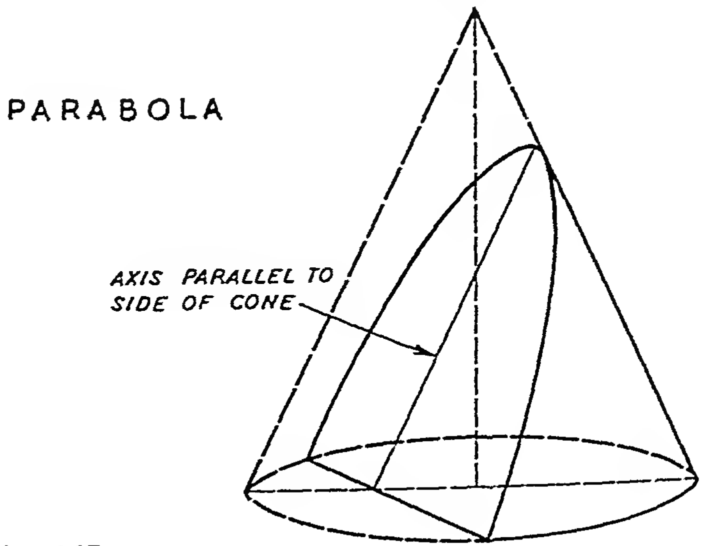


Fig. 167

axis, height and ordinates are given. AB is the ordinate, EC the axis and D the vertex or point where the curve cuts the axis. DC is made equal to ED. Lines AC and CB are drawn and each divided into the same number of units, and the points of division are indexed as shown. By joining corresponding divisions, intersections are made through which the curve can be drawn.

The foregoing methods can be used in drawing rampant parabolas in a similar manner to the drawing of rampant ellipses previously described.

Describing a Parabola for Practical Work

Fig. 169 shows how a parabola can be set out for practical work using a straight-edge and square. AC is the ordinate and DB the

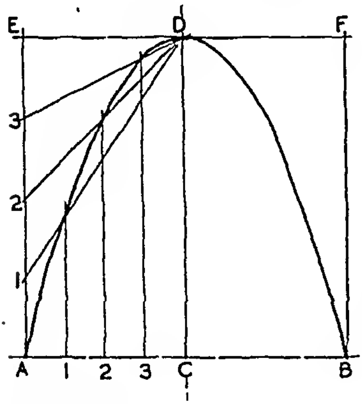


Fig. 168A

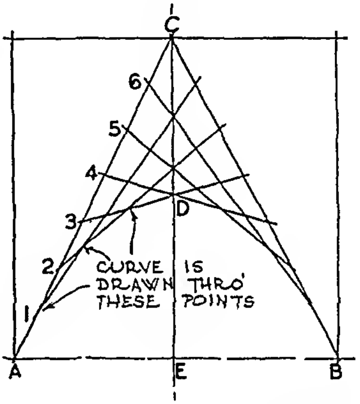


Fig. 168B

SETTING OUT OF PARABOLA

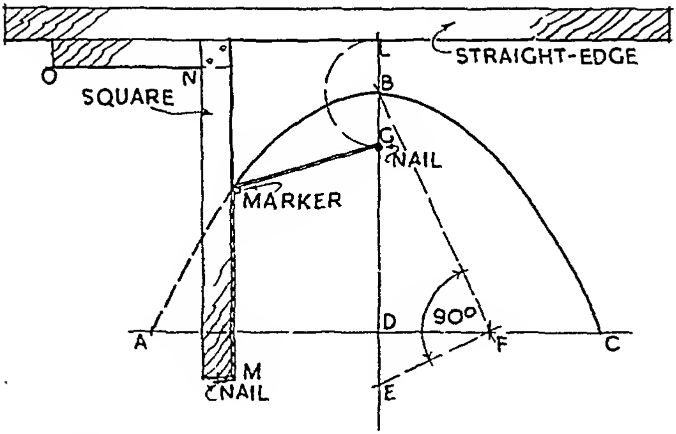


Fig. 169

axis; B is the vertex. DC is bisected in F, which is joined to B. FE is drawn at right-angles to BF to cut the axis BD produced in E. BG is set off equal to DE and G is the focal point of the parabola. BL is made equal to BG and the straight-edge is laid on L parallel to AC. A piece of string equal in length to LE is attached at one end to a nail at G and at the other end to M on the square MNO. By sliding the square along the straight-edge and at the same time keeping the string tight against the square with a pencil or marker the curve can be drawn.

To construct a parabola when the focus and directrix are given (Fig. 170):

Note: A parabola is the locus of a point which moves in a plane in such a way that its distance from a fixed point, the *focus*, is always equal to its distance from a fixed line, the *directrix*.

AB is the directrix. Bisect AB to find C and draw the perpendicular CD to AB. CD is the axis. F is the focus, therefore E, the vertex of the parabola, is midway between C and F. Plot any number of points from C along the axis, indexed 1 to 6 in this case, and through these points draw parallels to AB. With centre F and $C1$ as radius describe an arc to cut the parallel through 1; repeat this procedure with centre F, radius $C2$ and parallel through 2 and so on to find points on the required curve.

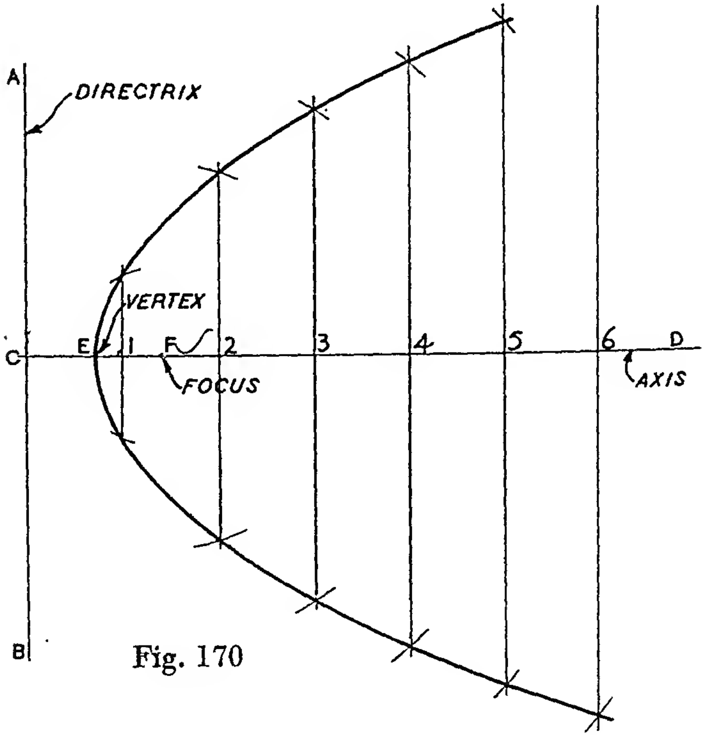


Fig. 170

THE HYPERBOLA

This curve is the outline of the section obtained when a cone is cut perpendicular to its base, but not through the centre of the base (Fig. 171A).

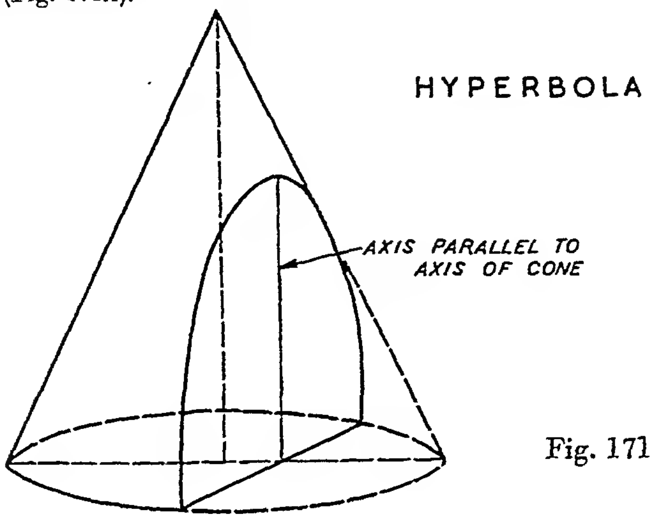


Fig. 171A

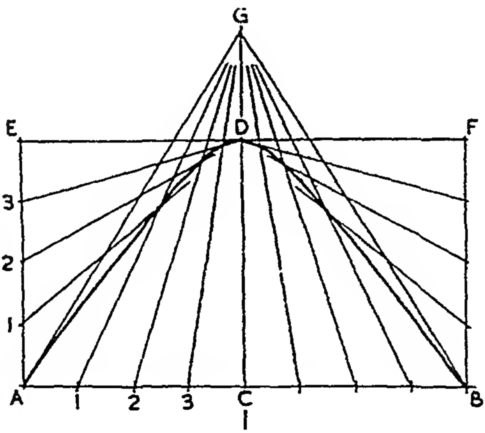


Fig. 171B

Setting out the Hyperbola

Fig. 171B shows a method of drawing a hyperbola when the axis, *CG*, the vertex, *D*, and the ordinate, *AC*, are given. On *AC* a rectangle, *ACDE*, is constructed. *AC* and *AE* are then divided into a convenient number of corresponding parts. From the divisions along *AC* lines are drawn to *G*, and from the

corresponding divisions along AE lines are drawn to D. Where intersections are made points are found through which one-half the required curve can be drawn as shown. The procedure can then be repeated to complete the whole curve.

Parallel Curves

If a number of circles of varying radii are drawn with a common centre they are said to be concentric circles and their curves are parallel to one another. But in the case of other figures, except those built up from the arcs of circles, curves drawn parallel to the true curve of the figure are not themselves wholly or partly similar figures.

Fig. 172 shows the drawing of lines parallel to the curve of a true ellipse, whose axes are AB and CD. The lines have to be plotted by drawing lightly a number of equal circles with radius equal to the required distance between the parallels and centres lying on the true ellipse. The lines are themselves not true ellipses.

Spiral Curves

A spiral curve is the locus of a point moving on a plane surface about a fixed point in such a way as to approach nearer to or recede farther from the fixed point in a regular manner.

Of the various spiral curves the following examples have been chosen as being of the greatest practical value.

Archimedean or Equable Spiral

Fig. 173 illustrates the principle of this curve. A is the fixed point or centre of the spiral and $A1$ the longest radius vector (i.e. the line from the fixed point to the farthest point of the locus). Lines $A2$, $A3$, $A4$, $A5$ are drawn, the angles formed at A being all equal. $A1$ is divided into 10 (or any number) equal parts and using compasses with centre A points are marked on $A1$, $A2$, $A3$, etc., at progressively shorter distances by one division from A. A free curve passing through these points is part of the required spiral curve. The name "equable" applied to this curve is self-explanatory; "Archimedean" is after Archimedes, a mathematician of Ancient Greece, who made much use of the properties of the spiral.

Fig. 174 illustrates the construction of an Archimedean spiral of two convolutions, i.e. one in which the moving point travels twice round the fixed point. OA is the longest radius vector and is divided into 24 parts. Other radius vectors are drawn so that twelve equal angular spaces are formed. Then, using compasses with centre O and radii progressively shorter by one division from

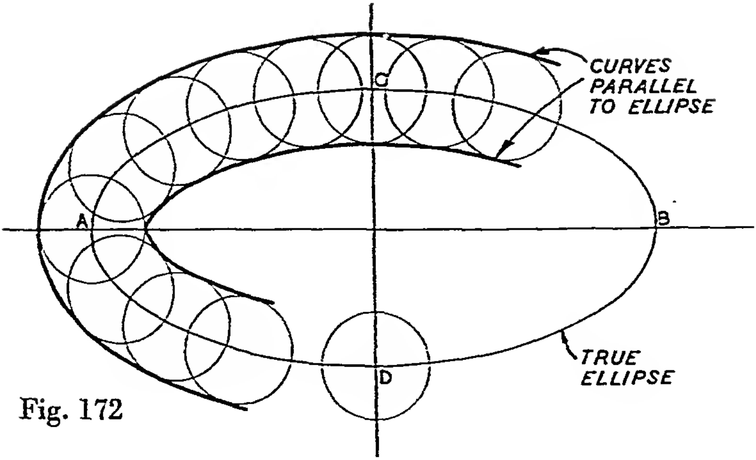


Fig. 172

EQUABLE SPIRALS

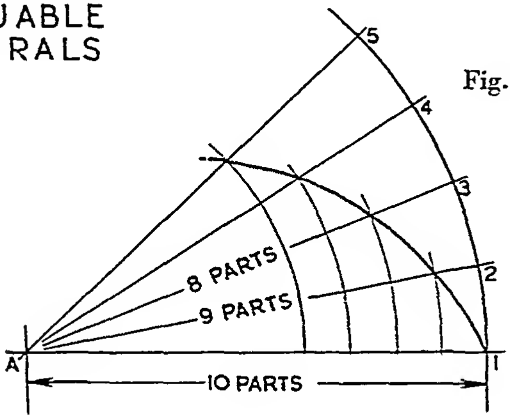


Fig. 173

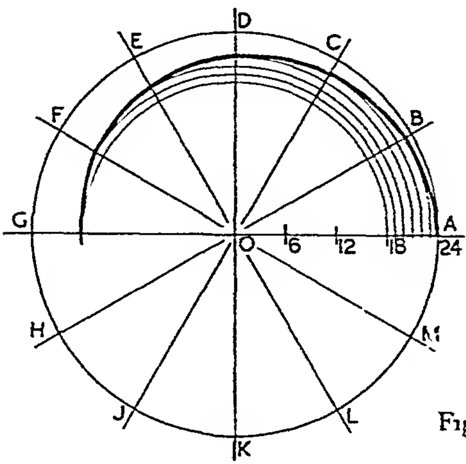


Fig. 174

A arcs are drawn to cut the vectors in turn. By joining the points so obtained in a smooth curve the required spiral is drawn.

(Note: Great care is required in drawing these freehand curves).

Fig. 175 shows a spiral of constant pitch built up of quadrants, the centres of which are located at the corners of a square. The pitch equals the perimeter of the square.

Fig. 176 shows the drawing of the involute of a circle. The procedure is as follows: Draw a small circle and divide the circumference into any number of equal parts, 12 in this case. At point *O* draw a line tangent to the circle and along it mark point 3′ os that 03′ equals a quarter of the circumference of the circle; divide 03 in three equal parts to find points 1 and 2, and set off equally points 4′ to 12′. Draw tangent lines to the indexed points about the circumference and index these lines correspondingly. With centre 0 and radius 0 12′ describe an arc to tangent 1, then with centre 1 describe an arc to continue the previous one to tangent 2, and so on to complete the involute.

The Helix

The helix is the locus of a point moving round the surface of a cylinder and at the same time advancing axially, the rate of progress being constant in both directions. The advance (axially) per revolution is known as the pitch.

Fig. 177 shows a cylinder with a helical line about the surface. The pitch of the helix and its development or falling line are found as follows: The circumference of the plan of the cylinder is divided into any number of equal parts (12 in this case) which are indexed. The axis of the cylinder is also divided into equal parts (12 in this case). Ordinates are projected from the points on plan to contact corresponding co-ordinates on the elevation, thus giving a series of tracing points for the location of the helix. Units of the divisions of the circumference are then plotted along the ground line of the elevation and perpendicular ordinates are produced to contact horizontal co-ordinates from the vertical divisions on the elevation, thus giving points of intersection through which passes the development of the helix, or falling line, which is a straight inclined line. If the diagram is regarded as representing a spiral (or turret) staircase, each unit space on plan can be made to correspond to a winder, and each unit rise on elevation to the riser of a step.

Fig. 178 shows the outlines of a handrail for such a staircase. A suitable number of winders are set out on plan, which shows the width of the handrail, and the height in elevation is divided into units corresponding to the risers of the required steps (storey

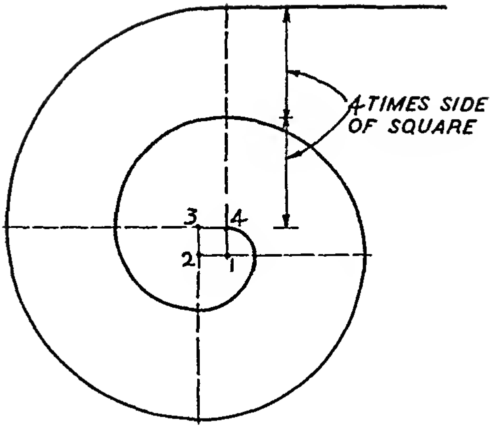


Fig. 175

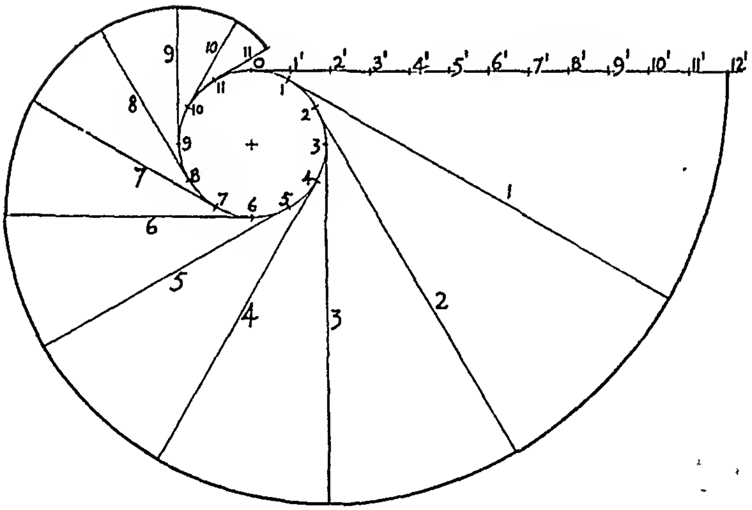
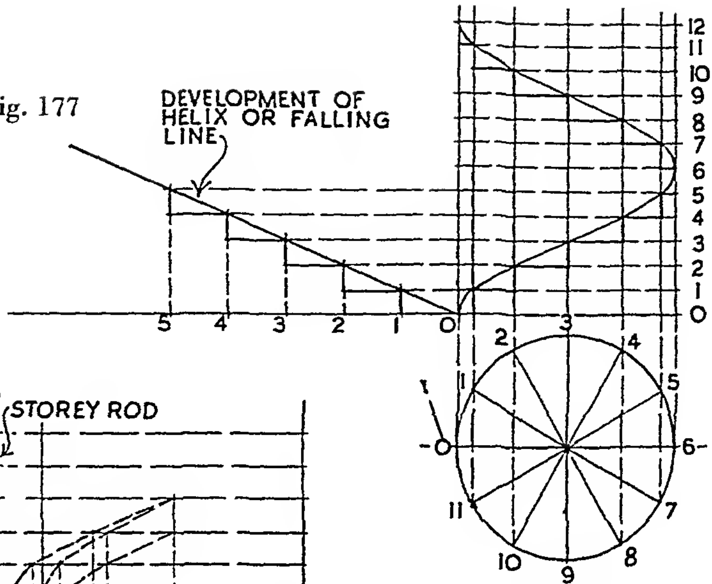


Fig. 176

Fig. 177



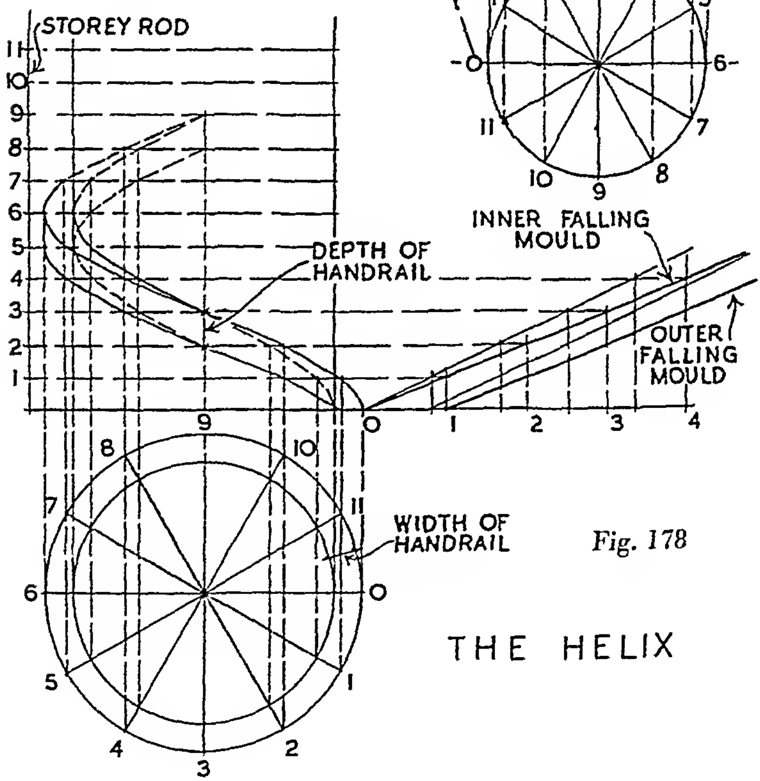


Fig. 178

THE HELIX

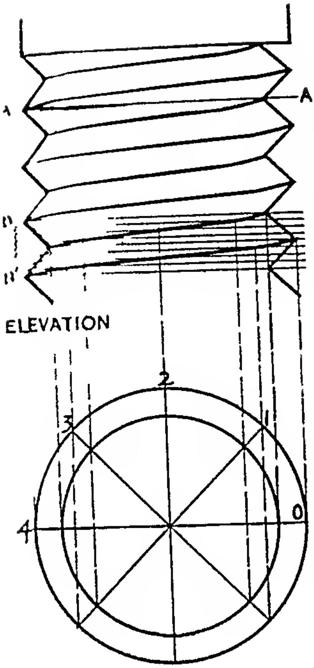


Fig. 179

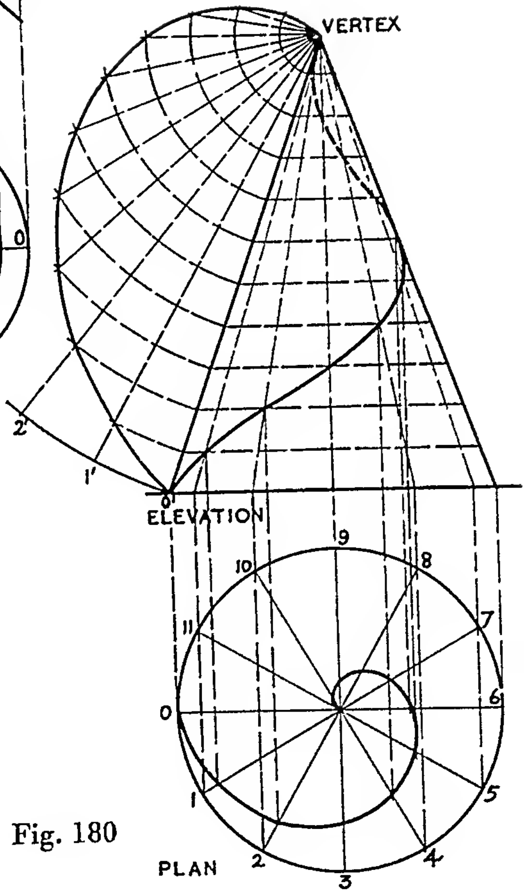


Fig. 180

rod). Ordinates and co-ordinates are projected as previously described and the elevational view and development are obtained.

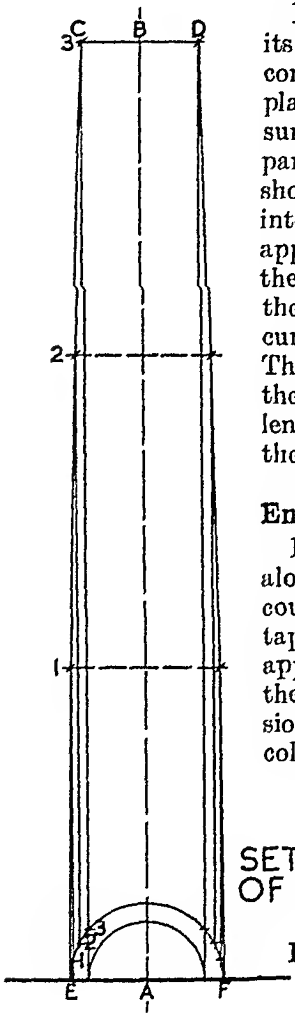
Fig. 179 shows a right-handed V-threaded bolt; another application of the same principal. The pitch of the thread is equal to *B–B*, and if line *A–A* is at right-angles to the axis of the bolt, then half a turn advances the thread half the pitch.

The Spiral

Fig. 180 shows a right cone with a spiral about its surface, and also the part development of the cone showing the course taken by the spiral. The plan and elevation of the cone are drawn and the surface is divided up into a number of equal parts by lines drawn from base to vertex as shown. The height in elevation is also divided into a number of equal divisions. By the application of the previously described principle the spiral is plotted in elevation, and then from the elevation projected to the plan. The curve on plan is an "Archimedean" spiral. The development of surface is obtained by using the vertex as centre with the radius equal to length of side of cone. Then plot about this arc the units 0 to 12 on the base of cone.

Entasis

Entasis is the swelling or curving outwards along the outline of a column shaft intended to counteract the optical illusion which gives a tapering shaft bounded by straight lines the appearance of curving inwards. It was used by the Classic Greeks, to convey a subtle impression of the weight-bearing function of the column.



SETTING OUT OF ENTASIS

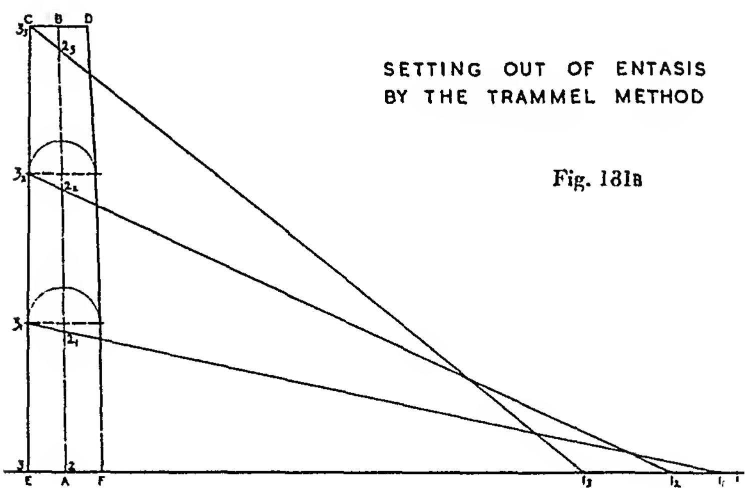
Fig. 181A

Method of Setting out Entasis

There are various methods, some giving an unpleasant exaggerated curve, but the one usually employed is illustrated in Fig. 181A. *AB* is the height of the shaft and *CD* and *EF* the top and bottom diameters respectively. With centre *A* and radii *AE* and *BC* two semicircles are drawn as shown. A perpendicular is erected from the limit of the smaller semicircle to cut the larger semicircle in 3. The segment *E*3 is divided into any number of equal parts—say three—and indexed from *E*. Through points 1 and 2 perpendiculars are erected to cut corresponding divisions in the height of the shaft. Through the points so obtained the required curve from *E* to *C* is drawn. The other side is drawn similarly.

A variation of this method is to draw the sides of the shaft perpendicular from the lower diameter for one-third the height and then to proceed by the method described above.

Fig. 181B gives a method more useful from a practical point of view by using a trammel. Let *AB* be the height of shaft. *CD* equals the top and *EF* the bottom. Extend the line *EF* in 1. It will be seen that the trammel is arranged at varied positions, thus 3_3, 2_3, is equal to 3, 2. The length of the trammel does not vary.



SETTING OUT OF ENTASIS BY THE TRAMMEL METHOD

Fig. 181B

CHAPTER XI

ARCHES

THERE are many kinds of Arches, employing many different curves. Similar curves also frequently serve as the outlines of domes and other features. The following are common examples of arches in brick and masonry:

1. *Flat* or *Camber Arch* (Fig. 182). The depth of such an arch is usually made a multiple of the depth of a brick course. The angle of the "skewback" should not be less than 60 degrees. The centre to which the joints radiate is thus determined. The camber of the soffit is normally $\frac{1}{8}''$ to every $1'\ 0''$ of span.

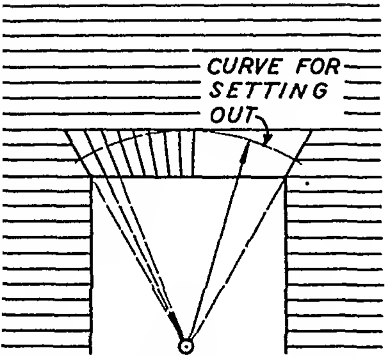
2. *Semicircular Arch* (Fig. 183). The depth of the arch need not be a brick dimension. All joints radiate from the centre of the semicircle.

3. *Segmental Arch* (Figs. 184 and 184A). Information concerning this arch will be found in Chapter VII, on Circles, although it is necessary that some terms relating to this and other arches should now be understood. It will be seen in Fig. 184A that the clear span of an arch is the distance between the walls by which it is supported. The Intrados, or soffit, is the underside of the arch, and the Extrados is the top or outer curve of the arch.

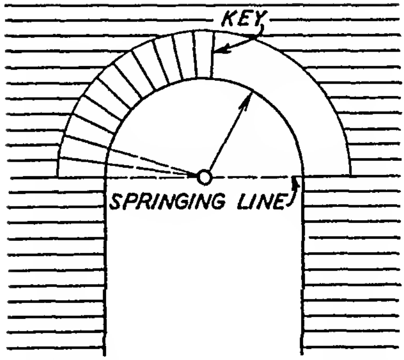
4. *Lancet* (*Gothic*) *Arch* (Fig. 185). The radius is greater than the span.

5. *Equilateral Arch* (Fig. 186). The radius is equal to the span.

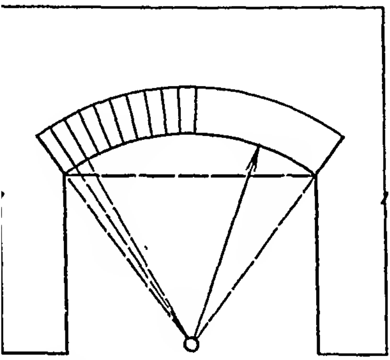
6. *Four-centred Arch* (Fig. 187). The span is divided into four equal parts, 1, 2, 3, 4, forming a square. From centres 1 and 2 the lower part of the arch is drawn, and from centres 3 and 4 the previous arcs can be continued to complete the arch.



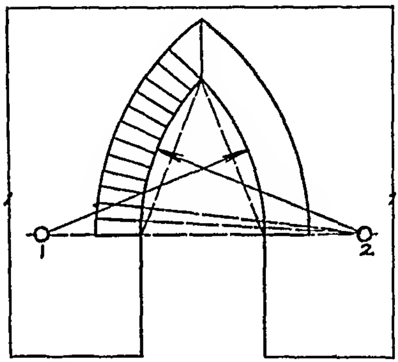
FLAT OR CAMBER Fig. 182



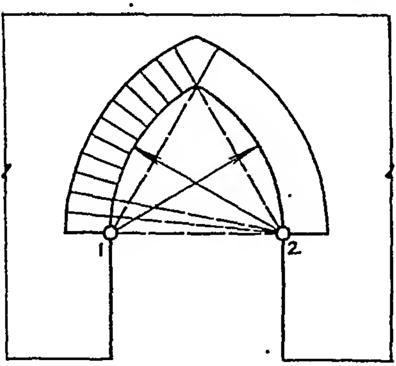
SEMI-CIRCULAR Fig. 183



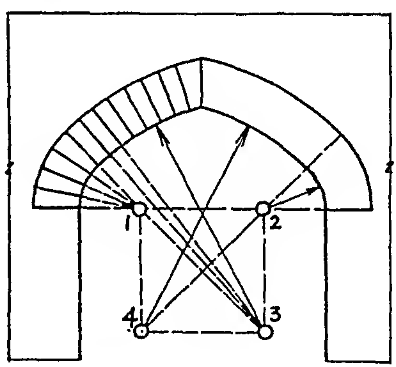
SEGMENTAL Fig. 184



LANCET Fig. 185



EQUILATERAL Fig. 186



FOUR-CENTRE Fig. 187

SETTING OUT OF ARCHES

7. *Circular Arch* (Fig. 188). All joints radiate to the centre of the circle.

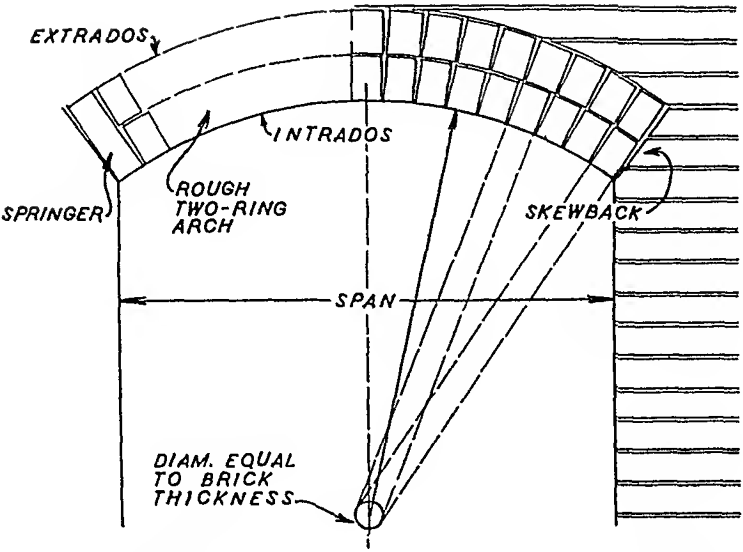
8. *Venetian Arch* (Fig. 189). The rise of the arch is greater than half the clear span, although not in any definite proportion. Also the depth of the arch is greater at the top than at the springing line.

9. *Florentine Arch* (Fig. 190). The intrados is semicircular, and the extrados is pointed, as in the previous example.

10. *Ogee Arch* (Fig. 191). The rise and span may be decided for good proportions thus: line *AB* will form the line of chords for the extrados, point *C* being the junction of the two chords. Erect the perpendicular bisector of *AC* and produce to find centre 3, and the perpendicular bisector of *CB* to find centre 1. These are the radii. Note the extrados must be drawn first.

11. *Rampant Arch* (Fig. 192). This arch is struck from two centres to form two quadrants of circles. In this case the smaller radius is half the dimension of the larger radius.

12. *Trefoil Arch* (Fig. 193). This arch is struck from four centres, each being equidistant from the centre line of the arch.



SEGMENTAL

Fig. 184A

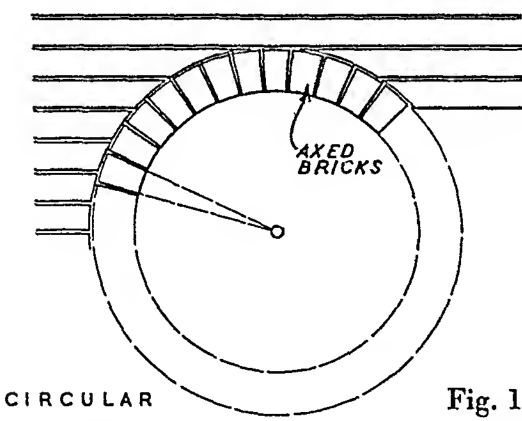
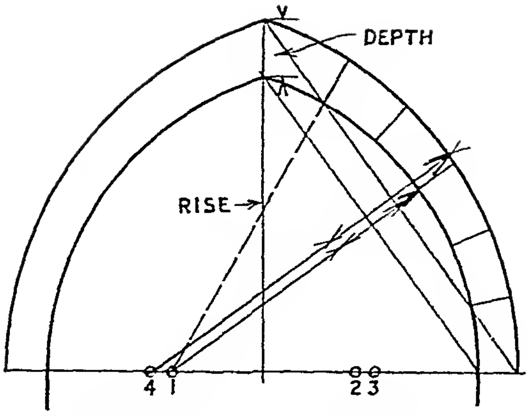
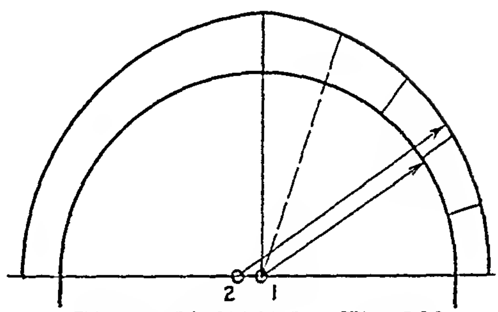


Fig. 188

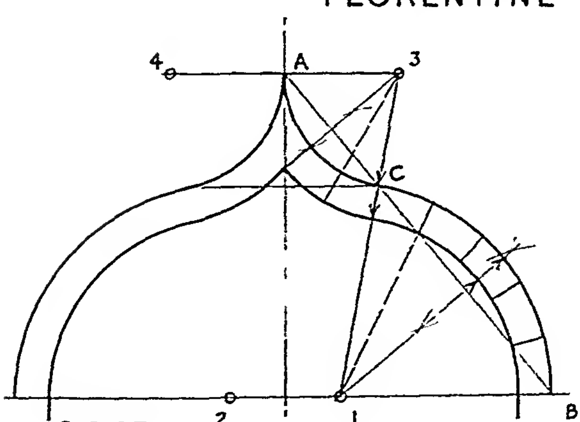
SETTING OUT OF ARCHES



VENETIAN Fig. 189



FLORENTINE Fig. 190



OGEE Fig. 191

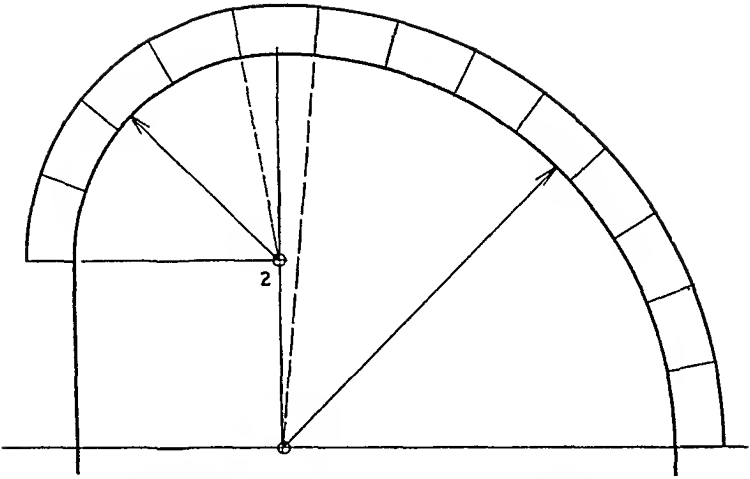


Fig. 192

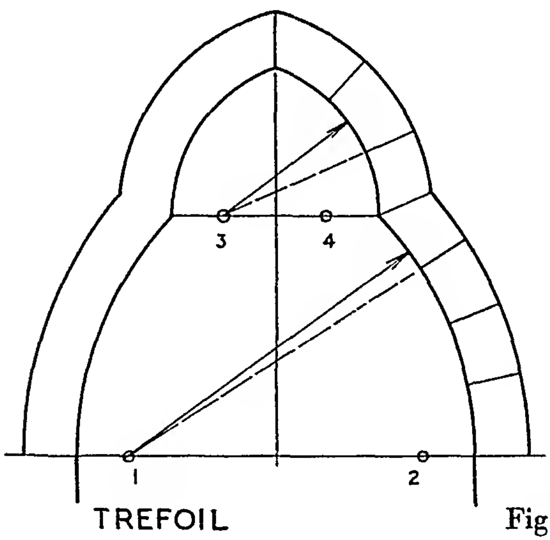


Fig. 193

CHAPTER XII

PATTERN

Geometrical Basis of Pattern and Ornament

Decorative designs, used for all manner of purposes, are often set out on a geometrical basis. Sometimes they consist entirely of combinations of straight and curved lines forming elementary figures of plane geometry. In making such designs it is always best to develop them from the existing "lines" and shapes of the object concerned. For example, the bonding of brickwork naturally suggests a diaper pattern composed by the arrangement of headers and stretchers. Similarly, a circular panel is appropriately filled with a concentric or radial pattern. In all cases, pattern and ornament in building work should never be applied without very careful consideration of its suitability and with regard to the nature, colour, texture, etc., of the materials involved.

Variations, even of strictly geometrical patterns are limitless. The designs shown here are intended to illustrate typical examples of the building up of patterns.

Continuous Band Patterns (Fig. 194):

A — Formed by regularly spaced perpendiculars joining parallel lines of any length.

B — Similar to A with the addition of diagonals.

C — Similar to A with the addition of semicircles.

D — Formed by regular circles and intersecting circles between parallel lines.

E — Combination of circles, perpendiculars and diagonals between parallel lines.

Repeating All-Over Patterns (Fig. 195):

These patterns can be extended indefinitely in any direction.

A — Built up from squares.

B — Built up from squares and diagonals.

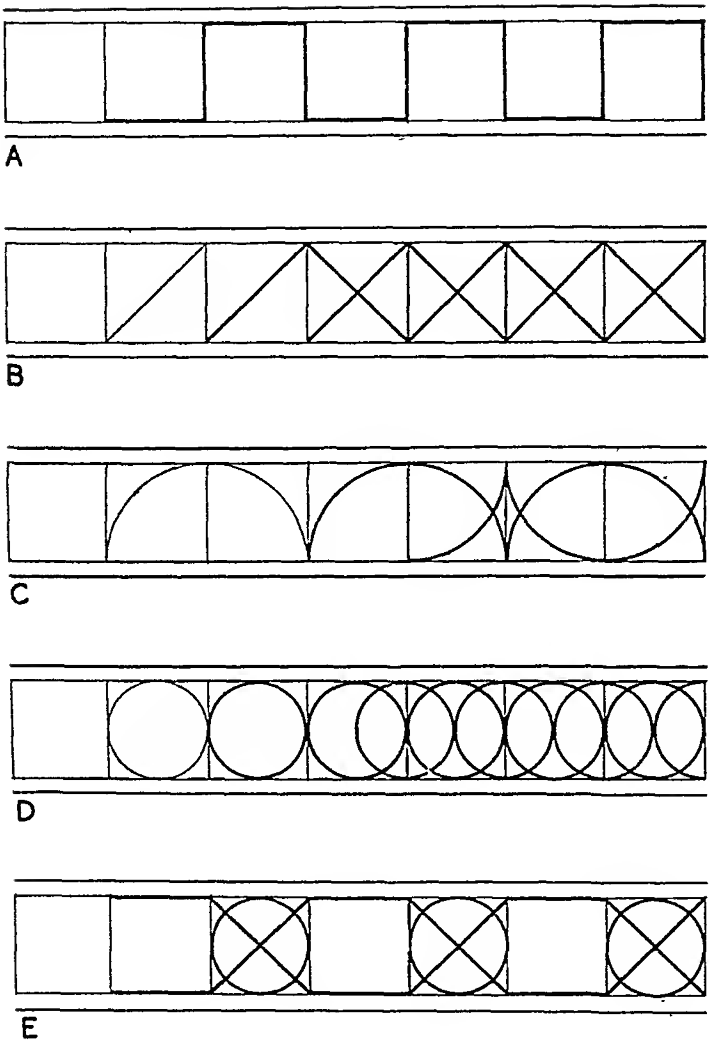
C — Built up from squares and circles.

D — Pattern built up from squares, diagonals and circles.

E — "Herring-bone" pattern.

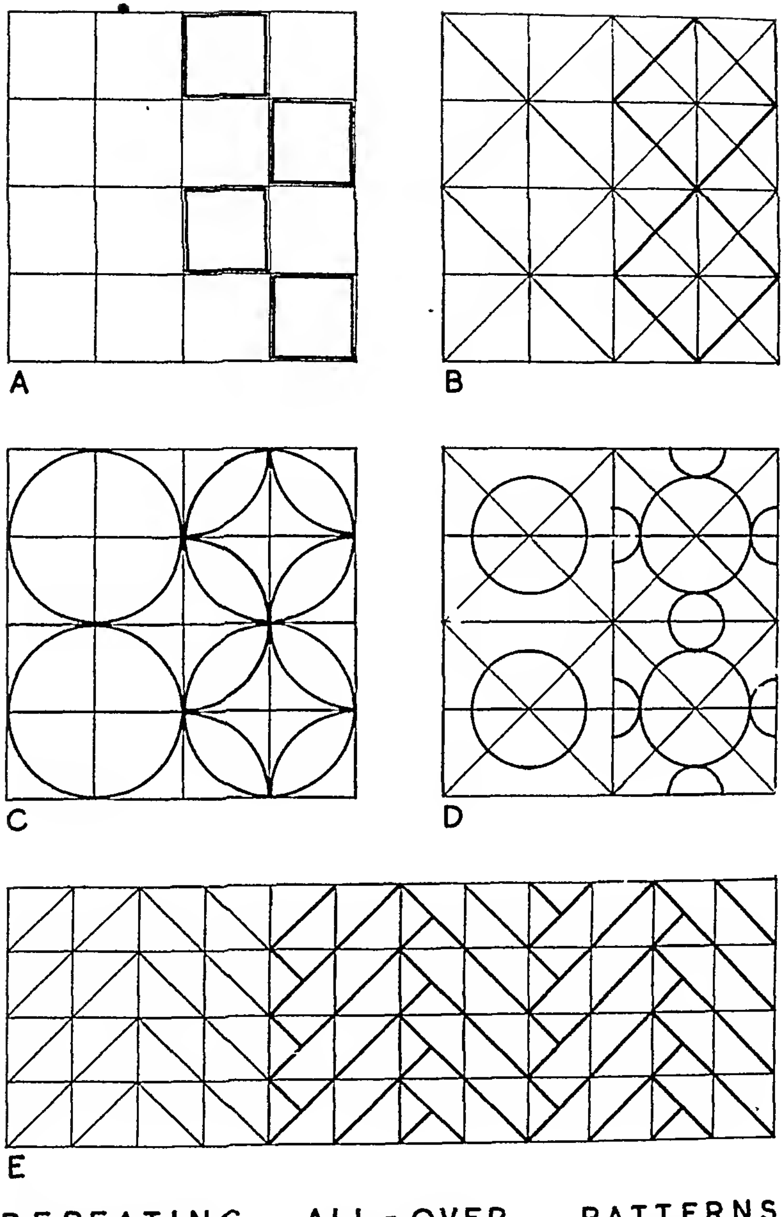
Patterns in Regular Areas (Fig. 196):

In designing a geometrical pattern to fill a regular area the basis should be the outline, axes, etc., of the shape, as illustrated by the squares, *A*, *B*, *C* and *D*, and the circles, *E* and *F*.



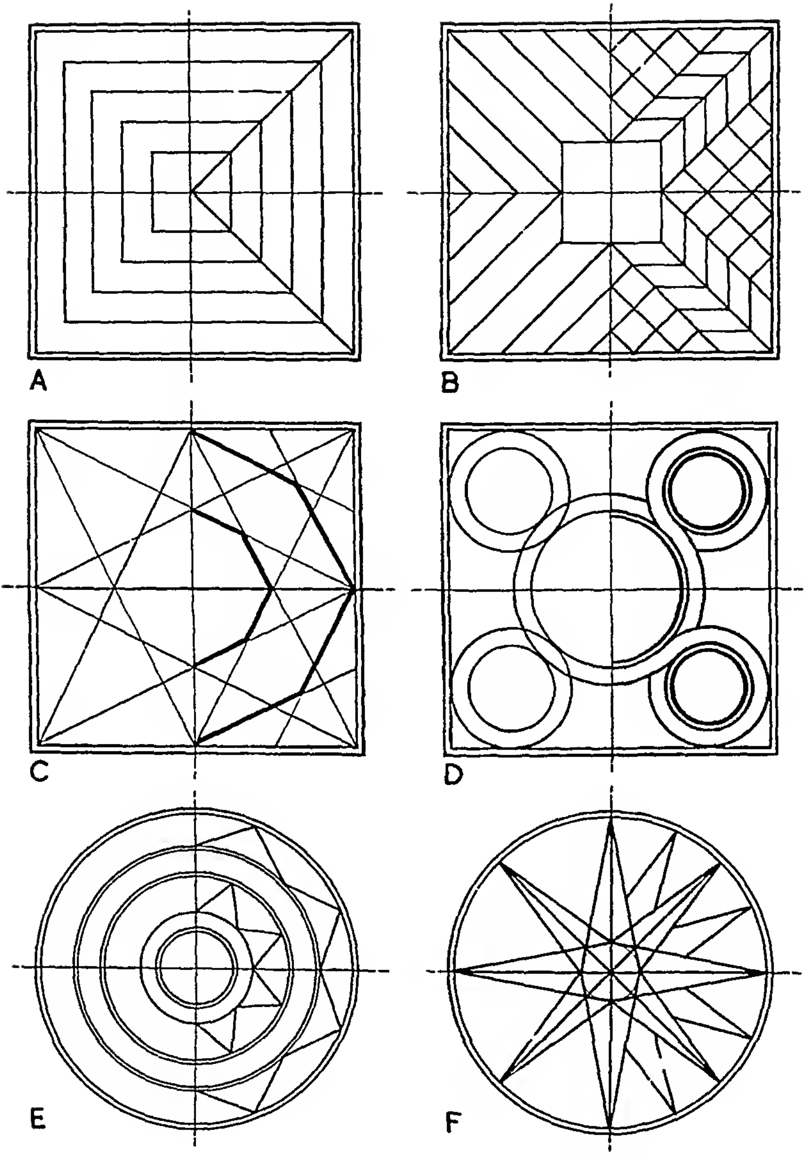
CONTINUOUS BAND PATTERNS

Fig. 194



REPEATING ALL-OVER PATTERNS

Fig. 195



PATTERNS IN REGULAR AREAS

Fig. 196

CHAPTER XIII

MOULDINGS

Setting out Intersections; Enlarging and Diminishing Mouldings

Mouldings, as elements of design, are less used in present-day work than they were in the past, and when used now are of comparatively simple profile. Nevertheless, occasions may arise when a knowledge of classic mouldings is required, particularly with regard to the restoration or extension of existing buildings. Figs. 197 and 198 show typical sections of "Greek" and "Roman" mouldings with their geometrical setting-out indicated. Other examples of common combinations of mouldings are shown in Fig. 199, an architrave, Fig. 200, base of a column, Fig. 201, simple classic cornice, Fig. 202, casement sash style.

Fig. 203 shows a more detailed setting out of a quirked ovolo moulding. The proportion is 7 parts deep by 6 parts wide, *AD* equals $1\frac{1}{2}$ units. Having set out the main lines, join *C* to *D* and bisect *CD* to find 0. Draw *OE* equal to 1 unit perpendicular to *CD*. With centre *E* and radius *EC* describe an arc *CD*. Divide this arc into a number of equal parts—8 in this case—and produce perpendicular ordinates to *CD* and continue as horizontals of corresponding lengths, i.e. 04 equals 04′. The curve of the moulding can now be drawn through the points so obtained.

Fig. 204 shows an alternative method of setting out a similar moulding. The proportion is 7 parts deep by 5 parts wide. From *D* draw lines to points 2, 3, 4, 5. From point 6 draw a line parallel to *D*2 to cut a vertical line from *D* at *C*. Produce *DC* and set off *CA* equal to *DC*. Divide *C*6 into 4 equal parts and through the points obtained draw ordinates from *A* to cut *D*3, *D*4 and *D*5. The curve of the moulding can now be drawn through the points so obtained.

Fig. 205 shows the drawing of the section of a scotia moulding. *AB* is the vertical height of the moulding. Set off the horizontal *BC* equal to half *AB*. Erect a vertical at *C* and set off *CD* equal to *BC*, and *DE* equal to two-thirds of *BC*. With centre *E* and radius *EC* describe a semicircle to *CE* produced at *F*. From *F* draw a line through *A* to cut the semicircle at *G*. Join *G* to *E*, cutting *AB* at *H*. With centre *H* and radius *HG* draw an arc to *A*, thus completing the required section.

MOULDINGS

GREEK ROMAN

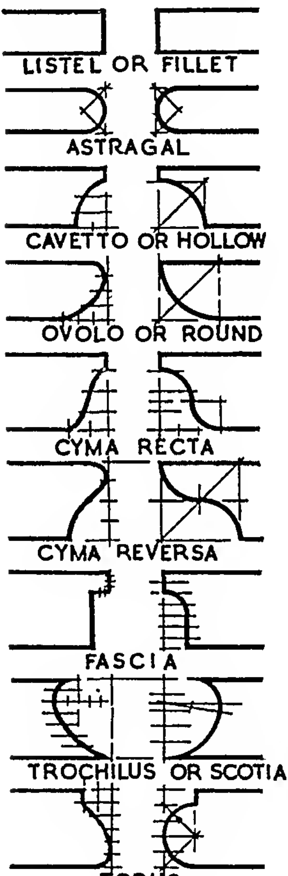
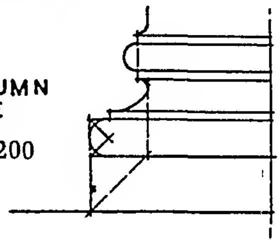


Fig. 197 Fig. 198

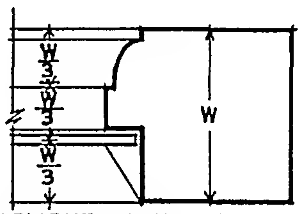
Fig. 199

ARCHITRAVE



COLUMN BASE

Fig. 200



CASEMENT SASH STYLE

Fig. 201

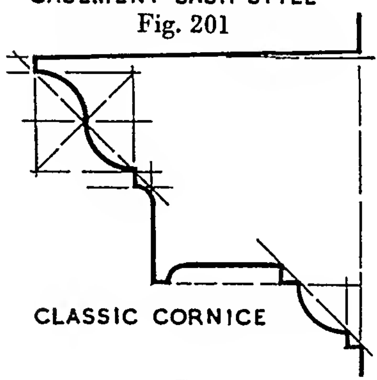


Fig. 202

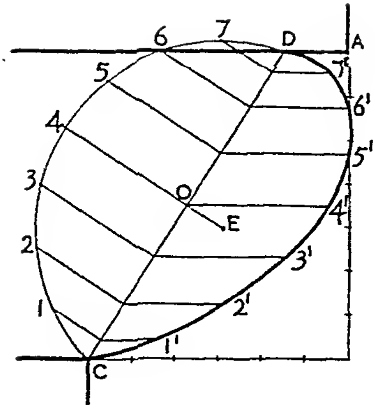


Fig. 203

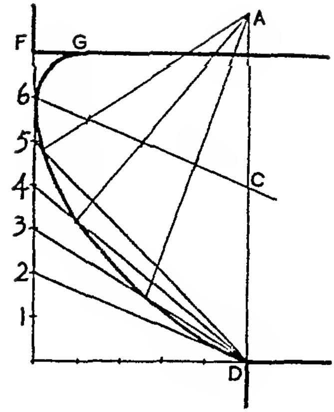


Fig. 204

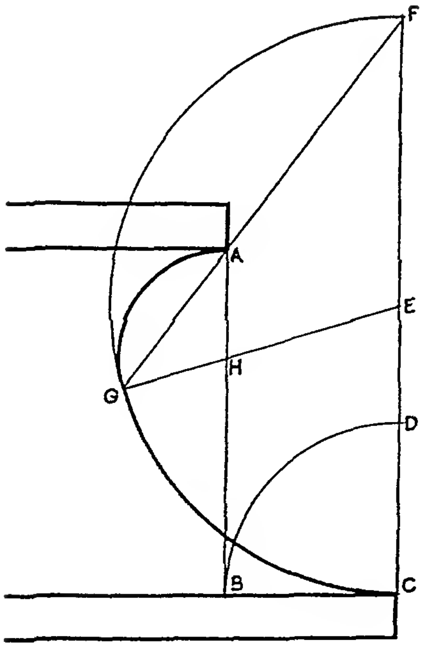


Fig. 205

Intersection of Mouldings

When any two mouldings intersect in the one plane, the junction is known as either (i) a bevelled mitre, or (ii) a mason's mitre, according to the position of the joint. In either case, it is possible to make the two intersecting mouldings of different widths and yet form a perfect mitre so long as the sectional detail corresponds.

Fig. 206 shows a $3\frac{1}{2}'' \times 1\frac{1}{2}''$ architrave moulding intersecting a similar moulding $2\frac{3}{4}'' \times 1\frac{1}{2}''$. Assuming the section of the larger moulding to be known, the section of the smaller moulding is found by projecting ordinates from the salient points of the former to the line of the mitre and thence at right-angles to contact corresponding co-ordinates projected from the side of the required

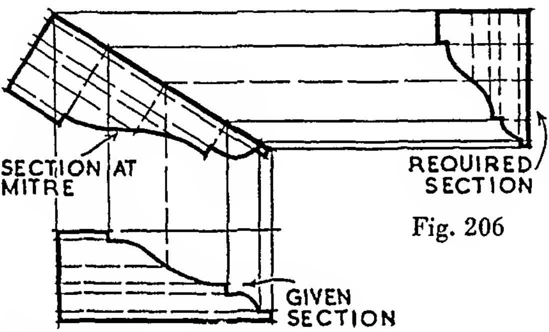


Fig. 206

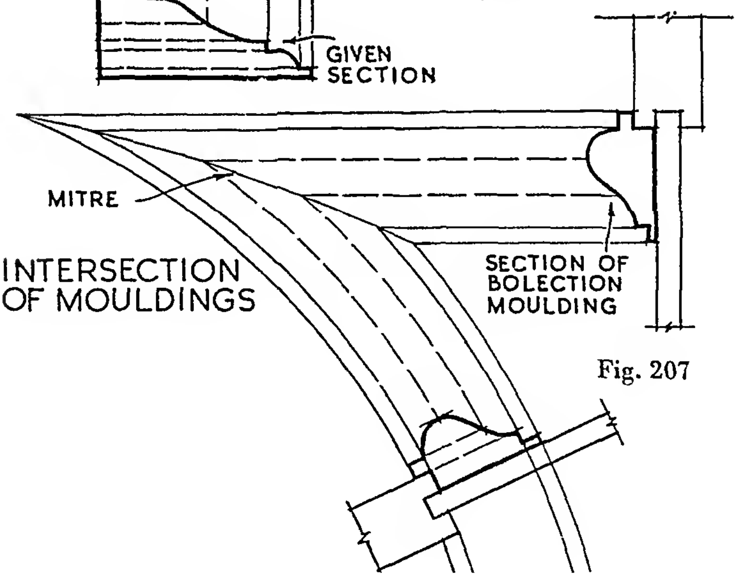


Fig. 207

section. The section at the mitre is obtained by projecting ordinates at right-angles to the line of the mitre to contact co-ordinates as before.

Fig. 207 shows the intersection of two bolection mouldings of the same section, one being straight and the other curved. The mitre is not a straight line, and is found by drawing through the intersections of lines from the salient points of the mouldings in the two directions.

Enlarging or Reducing Mouldings

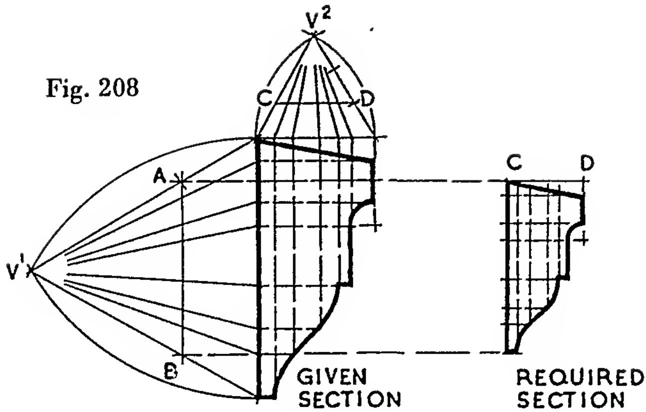
There are several methods of enlarging or reducing mouldings, or any drawing, such as by the use of the pantograph or proportional compasses, but the following illustrate the application of geometrical drawing.

Fig. 208 shows the full-size section of a moulding diminished to two-thirds its actual size. The full-size section is first drawn, and equilateral triangles are added as shown. The required lengths of the side and top of the moulding are marked within these triangles by the method indicated at AB and CD respectively, and by drawing lines from the salient points to V^1 and V^2 proportional reductions are obtained from which the required section can be plotted.

Fig. 209 shows how a moulded stair bracket, for example, can be reduced in width only for use at the winders of a geometrical staircase. The normal section is first drawn and a line AC, the length of the required section, is drawn at any angle. With centre A and radius AC an arc is described to line BA extended, and the proportional reductions of the moulding are produced in the same way. The required section can then be plotted as shown.

The above methods can be adapted for enlarging mouldings.

Fig. 208



ENLARGING AND REDUCING MOULDINGS

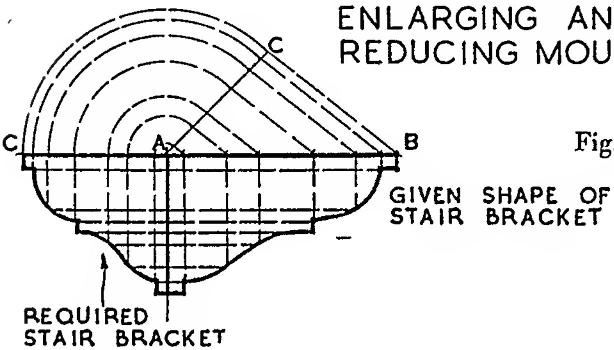


Fig. 209

CHAPTER XIV

ORTHOGRAPHIC PROJECTION

Representation of Solids in Plan, Elevation and Section; Projection of Building Details

The usual method of showing solid objects, which are actually three-dimensional, in two dimensional drawings is by means of related views termed *plans*, *elevations* and *sections*.

While the theory is difficult to explain briefly in words it is readily understood when a drawing in orthographic projection is seen. Figs. 210 and 211 illustrate the general principle. In Fig. 210 is shown a rectangular prism; parallel to its sides are three co-ordinated planes (planes at right-angles to one another), they are labelled: horizontal plane, vertical plane 1 and vertical plane 2. If the sides of the prism are projected at right-angles on to these planes they appear thereon as rectangles. Now, if it is imagined that the co-ordinate planes are "hinged" then, by swinging the horizontal plane downwards through an angle of 90 degrees and vertical plane 2 outwards through an angle of 90 degrees, the three projections will all lie in the same plane and the result is as shown in Fig. 211, which is an orthographic projection of the prism giving its plan and two related elevations.

In actual practice, of course, the projection would be made by drawing first the plan, then the elevation of the front face immediately above it, and then the end elevation at the side.

There are a number of variations of orthographic projection according to the relationship of the views obtained,[1] but it is usual in this country for drawings of buildings to be made so that, as shown in Fig. 211, the plan is the view obtained by looking down on the object, the front elevation by looking at the front face of it, and the end elevation by looking at the end nearest vertical plane 2. According to the shape of the object or the design of the building, several plans, elevations and sections may be required to show it fully, but, so far as possible, the above relationship of the views should be maintained; front elevations should be above plans and should line through horizontally with end elevations and sections.

[1] For a more detailed description, see *Draughtsmanship*, by R. Fraser Reekie. Arnold and Co., London.

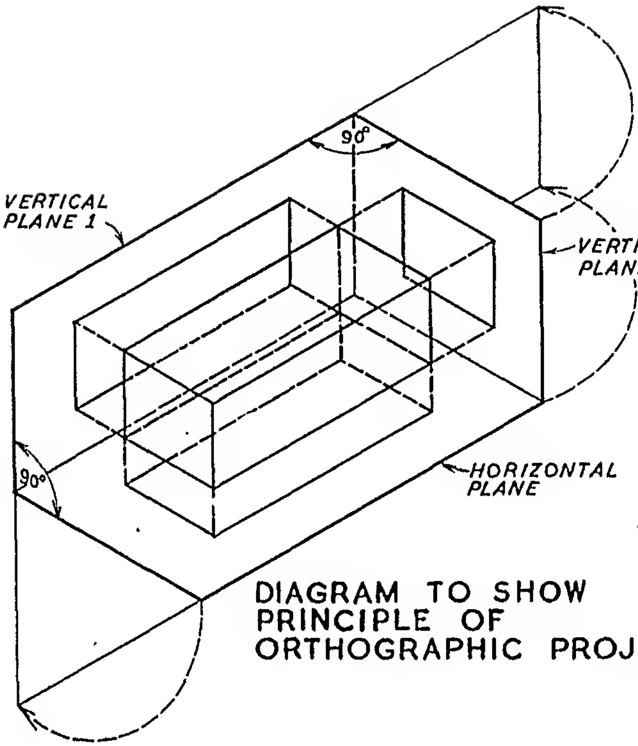


Fig. 210

DIAGRAM TO SHOW PRINCIPLE OF ORTHOGRAPHIC PROJECTION

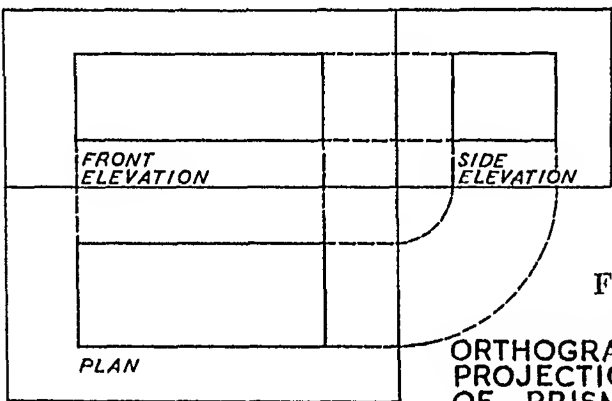


Fig. 211

ORTHOGRAPHIC PROJECTION OF PRISM

Fig. 212 illustrates various sections of the prism, assuming it to be hollow. If cut horizontally the view would be as in diagram 1; this view, properly termed a *horizontal section*, is usually called a plan, and when reference is made to the plan of a building it is usually the horizontal section which is meant. If cut longitudinally and vertically, diagram 2, the view is termed a *longitudinal* or *long section*. If cut across and vertically, diagram 3, the view is termed a *cross section*.

Fig. 212

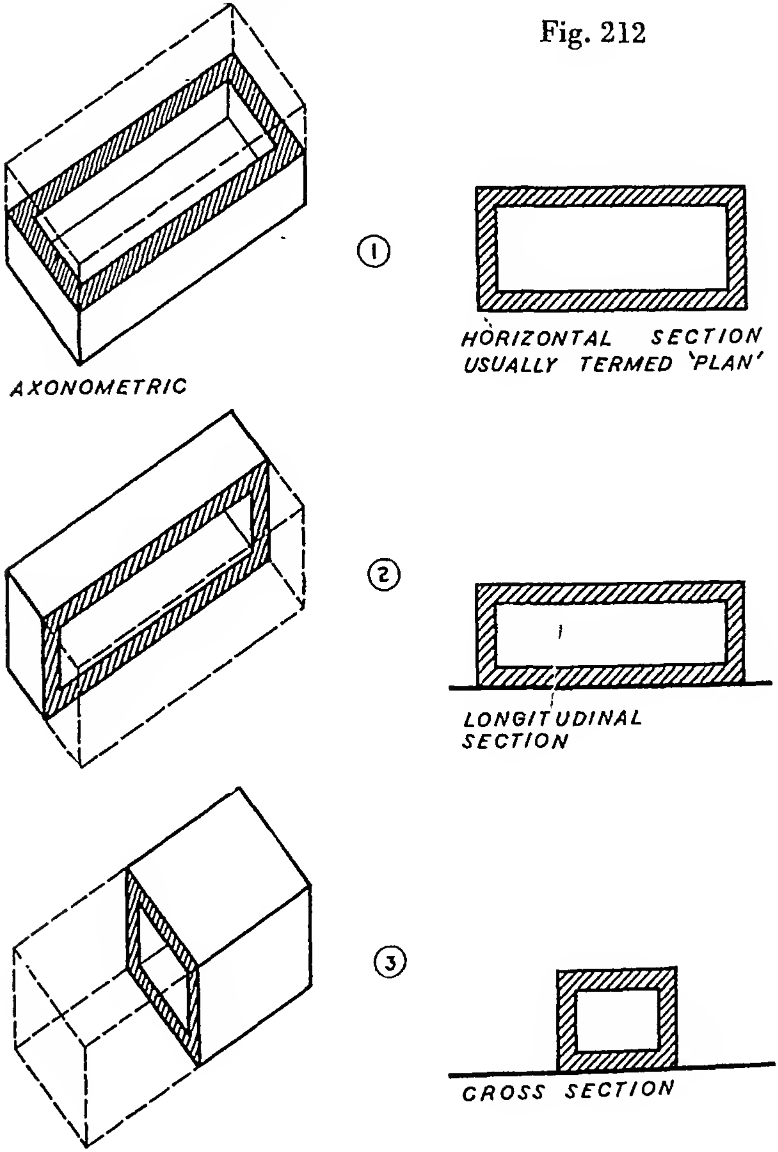


Fig. 213 shows simple geometric solids represented by orthographic projection in plan, elevation and side elevation. Pictorial views are also given so that the actual forms can be understood.

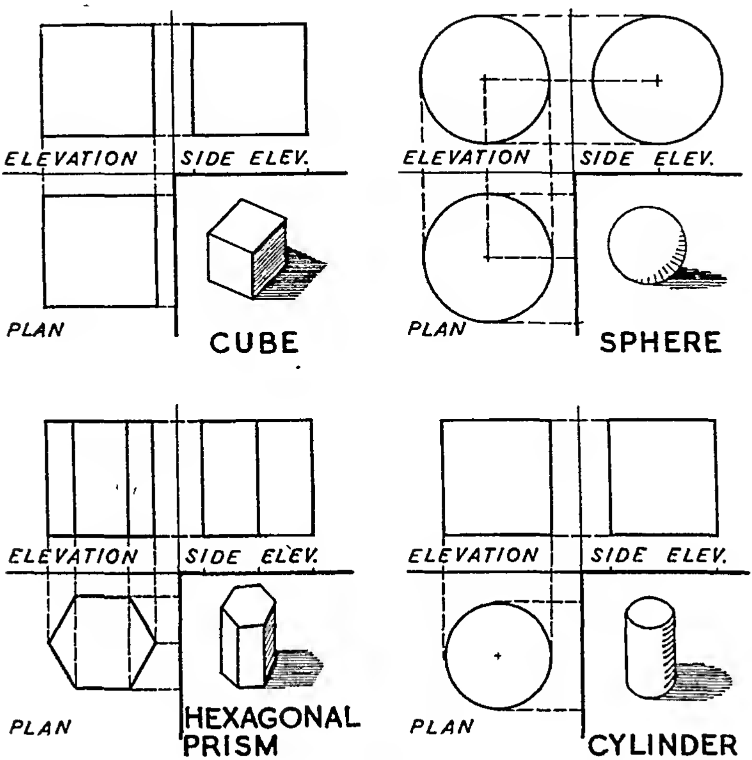
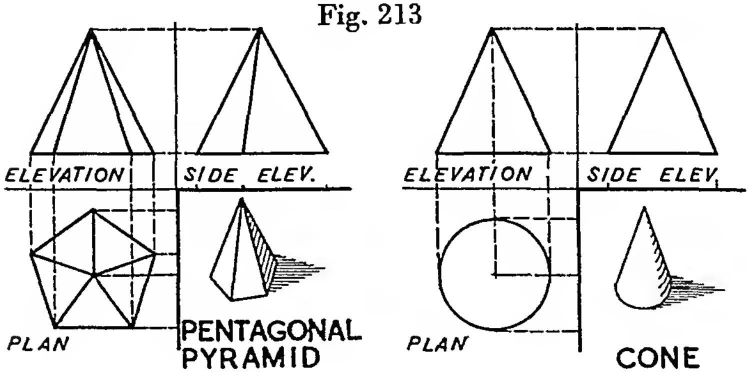


Fig. 213



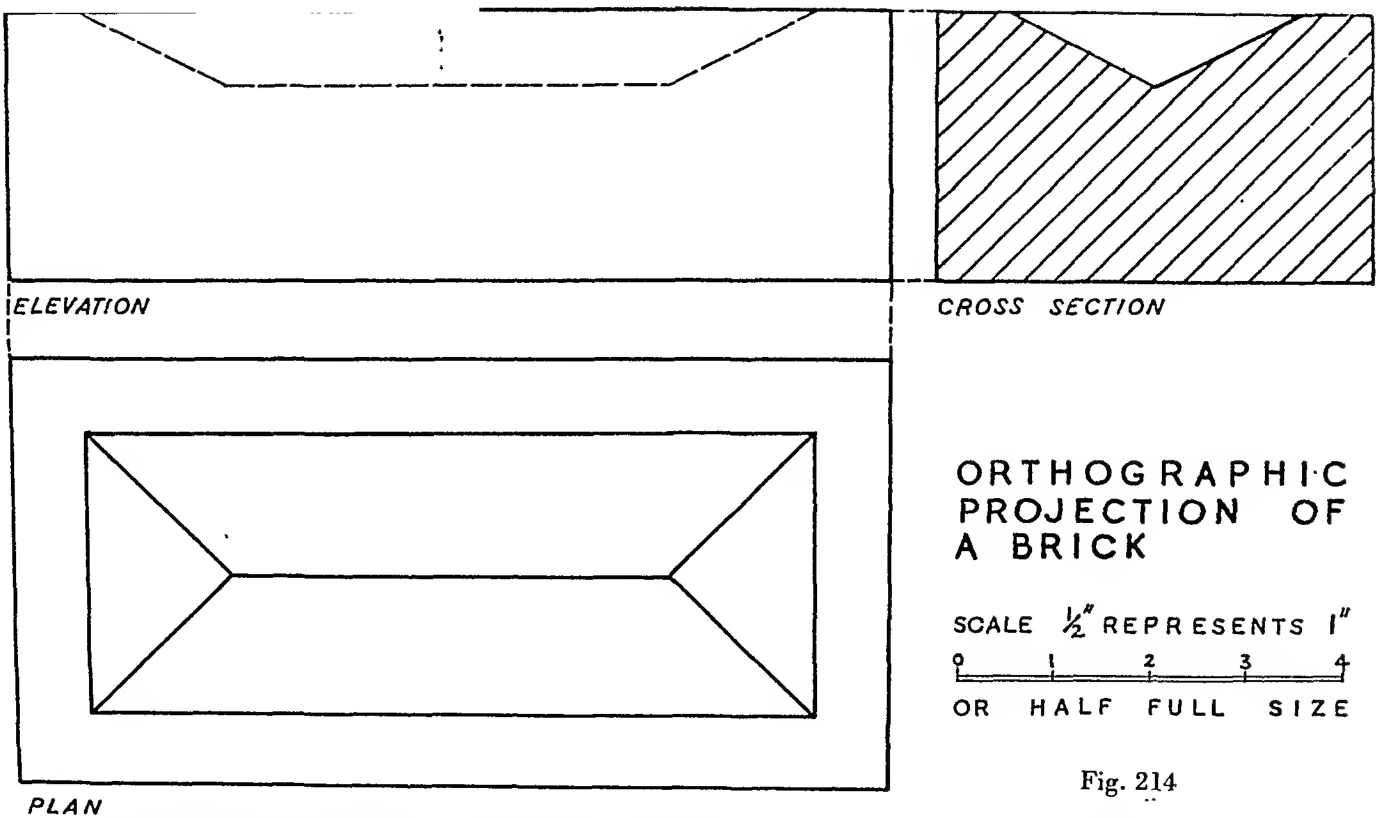


Fig. 214

Fig. 214 is a drawing of a brick, 9" long, $4\frac{1}{2}$" wide and $2\frac{3}{4}$" deep, in orthographic projection consisting of plan, elevation and cross section to a scale of ½" represents 1" or half full size. Note how the "frog" of the brick is represented.

CHAPTER XV

PICTORIAL PROJECTIONS

METRIC, AXONOMETRIC, ISOMETRIC AND OBLIQUE PROJECTIONS; PERSPECTIVE

Metric Projections

METRIC projections are methods of drawing objects or buildings so as to give an impression of actual three-dimensional appearance, yet in such a way as to allow lengths, breadths and heights to be measured. They are set up from orthographic projections and can be drawn to various scales. The metric projections most used are: Axonometric, Isometric and Oblique.

Axonometric Projection

This has the advantage of containing a true plan and is therefore more readily set up from existing drawings. Fig. 215 illustrates the principle. It is generally easiest to use the tee-square and 45 degree set-square, although as long as the "plan" view remains a true plan, the angle at which it is tilted to the horizontal can be varied. Construction lines are shown as broken lines in the two smaller examples to show how the setting up is made. Note that true circles on plan appear as true circles in the axonometric, but true circles in elevation becomes ellipses in the axonometric, and in setting up such shapes it is necessary to enclose them in rectangular "frameworks" of lines in order to plot their salient points.

Isometric Projection

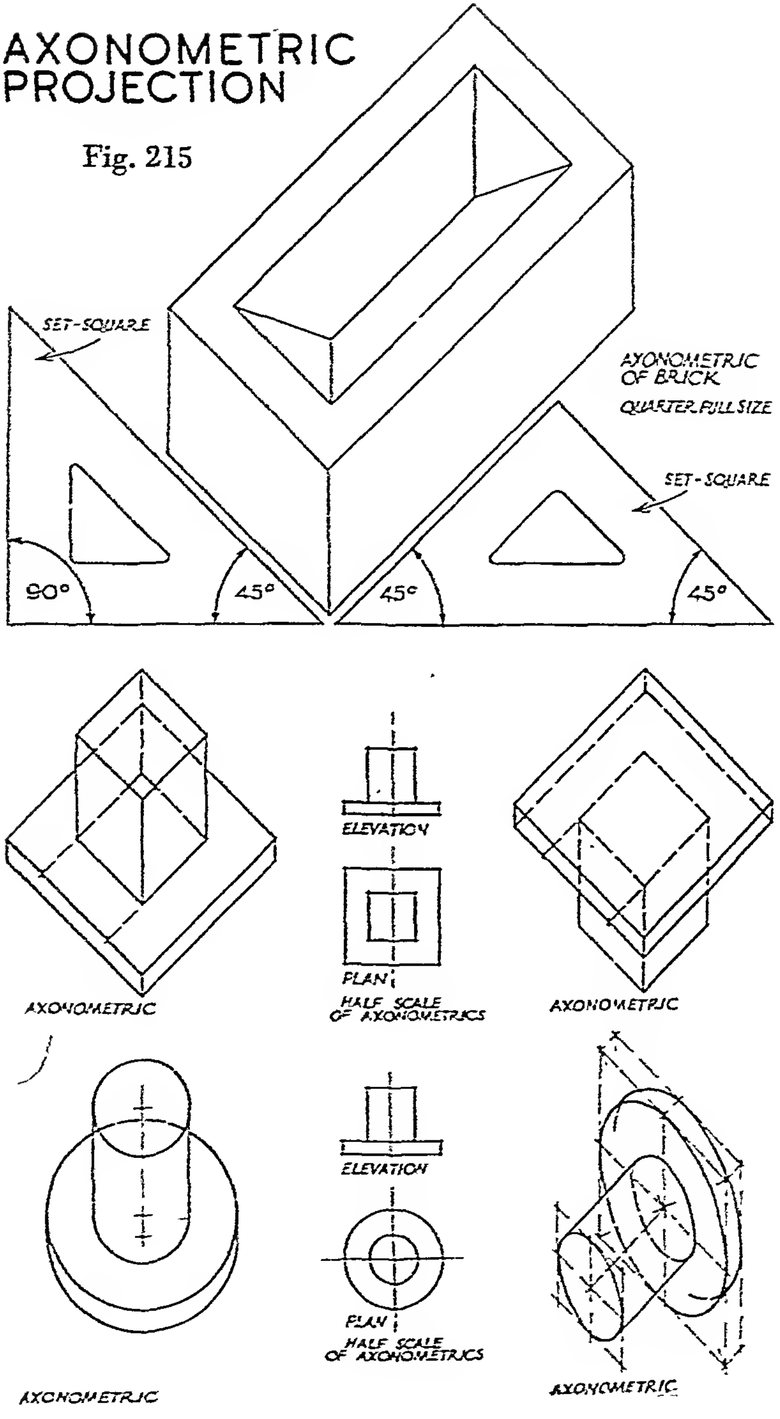
This is somewhat similar to axonometric, but the plan view is distorted. However, for some shapes a realistic effect can be obtained. Fig. 216 shows the principle and how the drawing is done using tee-square and 30-degree set-square. Circles appear as ellipses in both plan and elevation in the isometric, and have to be plotted as described above.

Oblique Projection

The principle is illustrated by the drawings in Fig. 217. There are two varieties of the method: (1) the oblique lines are drawn at 45 degrees to the horizontal and distances along them are at half the scale of that used for the horizontal and vertical lines; (2) the oblique lines are drawn at 30 degrees to the horizontal and the

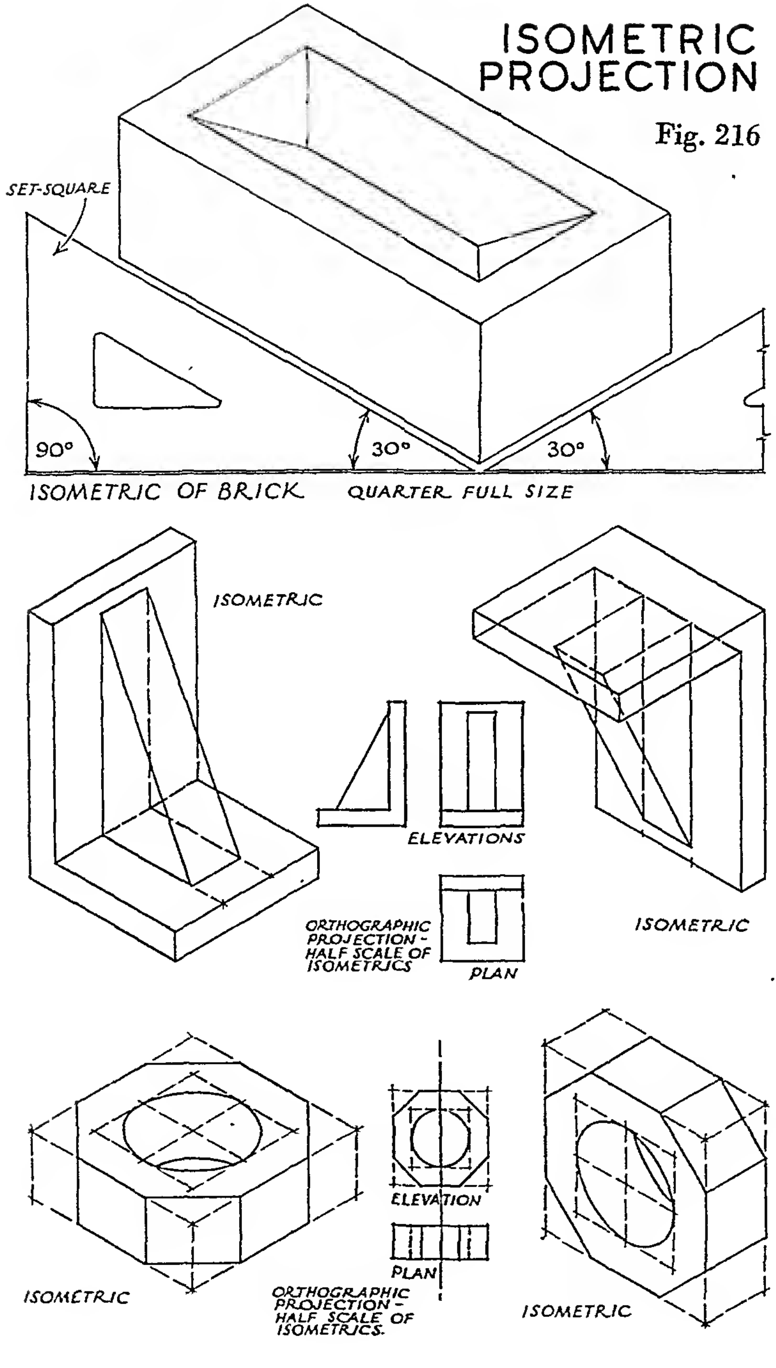
AXONOMETRIC PROJECTION

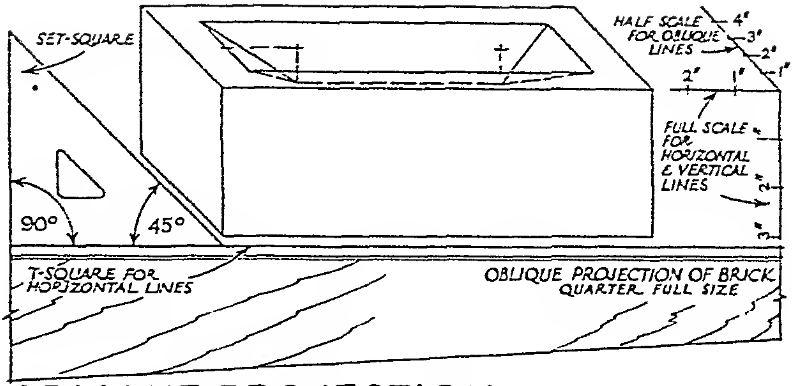
Fig. 215



ISOMETRIC PROJECTION

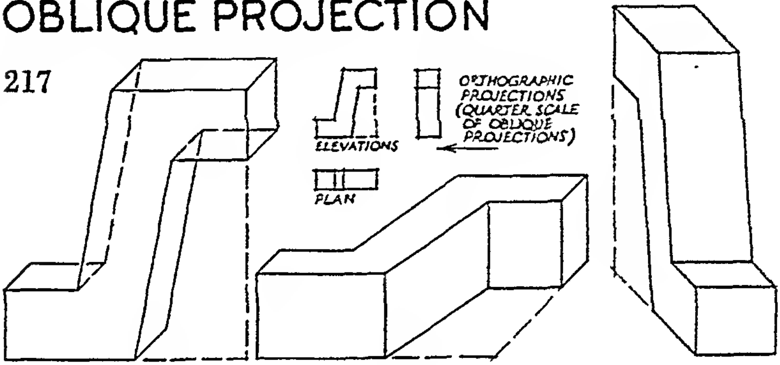
Fig. 216



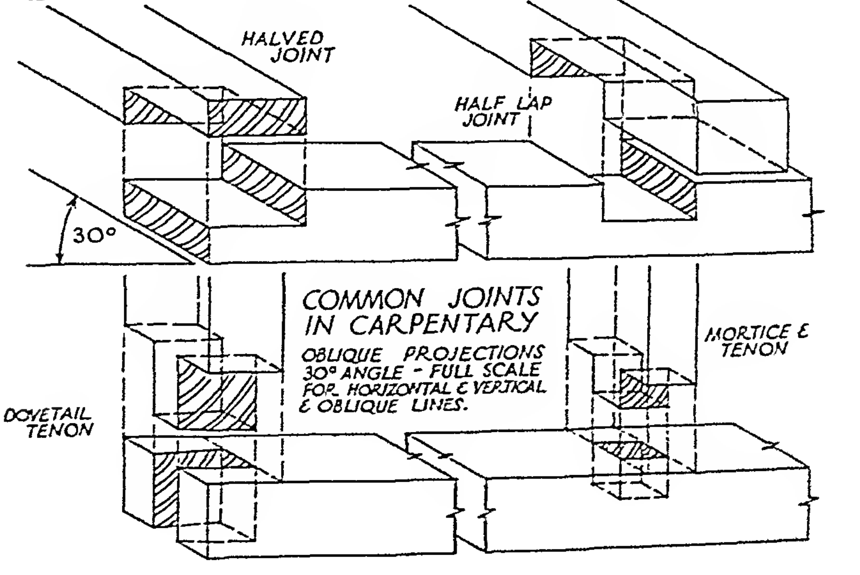


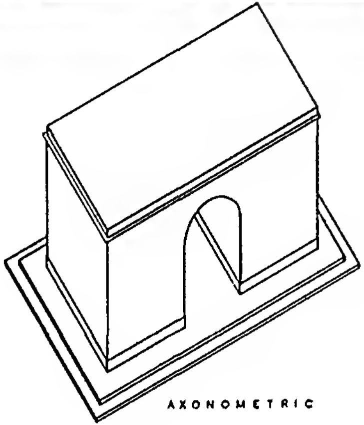
OBLIQUE PROJECTION

Fig. 217

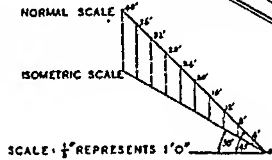


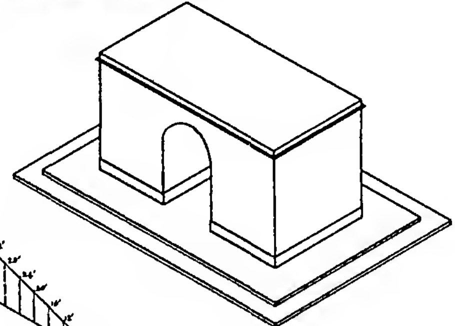
THREE DIFFERENT OBLIQUE VIEWS OF SAME OBJECT.



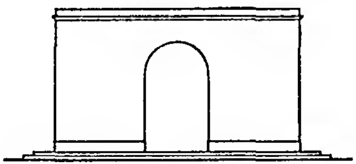


AXONOMETRIC

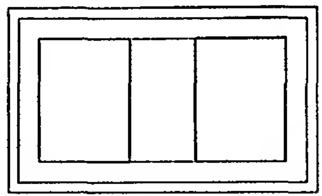




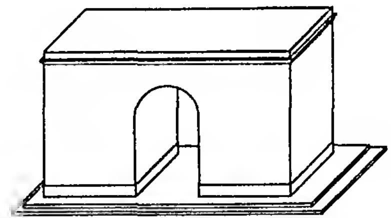
ISOMETRIC



ELEVATION



PLAN



OBLIQUE

NAME	METRIC PROJECTION	DATE

same scale is used for oblique, horizontal and vertical lines. Oblique projection is used chiefly for constructional details.

Perspective Projection

The chief use of a perspective drawing is to illustrate the actual appearance of an object or building, particularly in order to convey an impression of it to another person.

There are a number of methods of making perspectives, all based on the mechanics of human vision. It is not intended here to deal with the theory of the subject, but to explain what is considered to be one of the simplest and most satisfactory methods for all ordinary uses.

Fig. 218 shows a rectangular block drawn in perspective. The plan of the block is first drawn together with the elevations of the two sides which are to appear in the perspective, all to the same scale. A point, S, representing the position of the "eye" of the spectator, is next established in relation to the plan according to the view required, the precise position being determined in the light of experience or by trial and error. Lines are then drawn from the extremities of the plan, in this case FE, BC, to the point S, thus forming what is known as the angle of vision, which in general should not be more than 60 degrees nor less than 40 degrees. This angle is bisected and the bisector is the direct line of vision to which a line VP^1—VP^2, representing the picture plane, is drawn at right-angles. (The picture plane can be visualised as a vertical plane on which is projected the image of the block as seen by the eye of the spectator.)

The picture plane can be drawn in front of, through or behind, the plan according to the required size of the picture, which will be larger the farther the picture plane is taken from the spectator. In this example the picture plane is taken a little behind the plan, and to it from S are drawn lines which pass through the salient points, i.e. the corners, AD, BC, FE, GH. Lines are also drawn parallel to the sides of the block from S to contact the picture plane at VP^1 and VP^2 forming, of course, an angle of 90 degrees at S. A line in continuation of one side of the block, AD, FE, is also drawn to contact the picture plane at K.

At a reasonable distance above the drawing already made, a line parallel to the picture plane is drawn. This is the eye level in the picture. It coincides in normal vision with the horizon. At some estimated distance below the eye level, again according to the required picture, another parallel line is drawn. This is the base line or, in the case of a building, the ground line.

Perpendicular lines from the picture plane are now projected up to give the points VP^1 and VP^2 on the eye level—these are

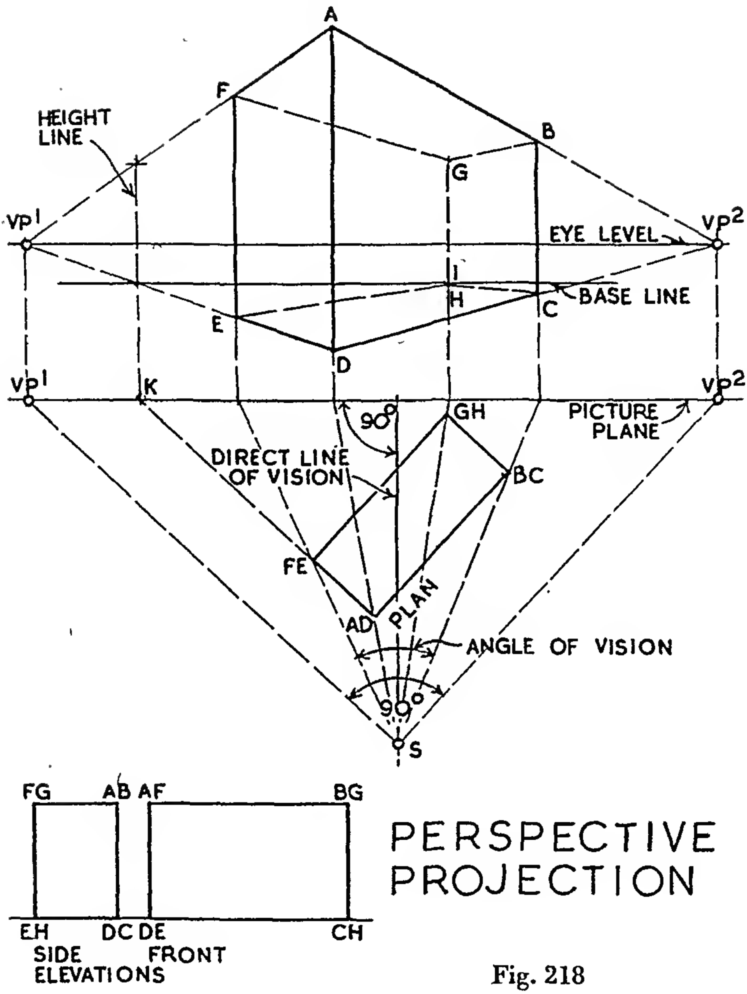


Fig. 218

the vanishing points to which lines along the sides of the block will converge in the picture. Perpendicular lines from the points on the picture plane are also projected up to give the height line from *K* and the vertical lines of the block.

The height of the block measured from the elevation is now marked up the height line from the base line, and then by drawing through the points so obtained to the appropriate vanishing points the tip and bottom lines of the block are obtained and the figure can be completed.

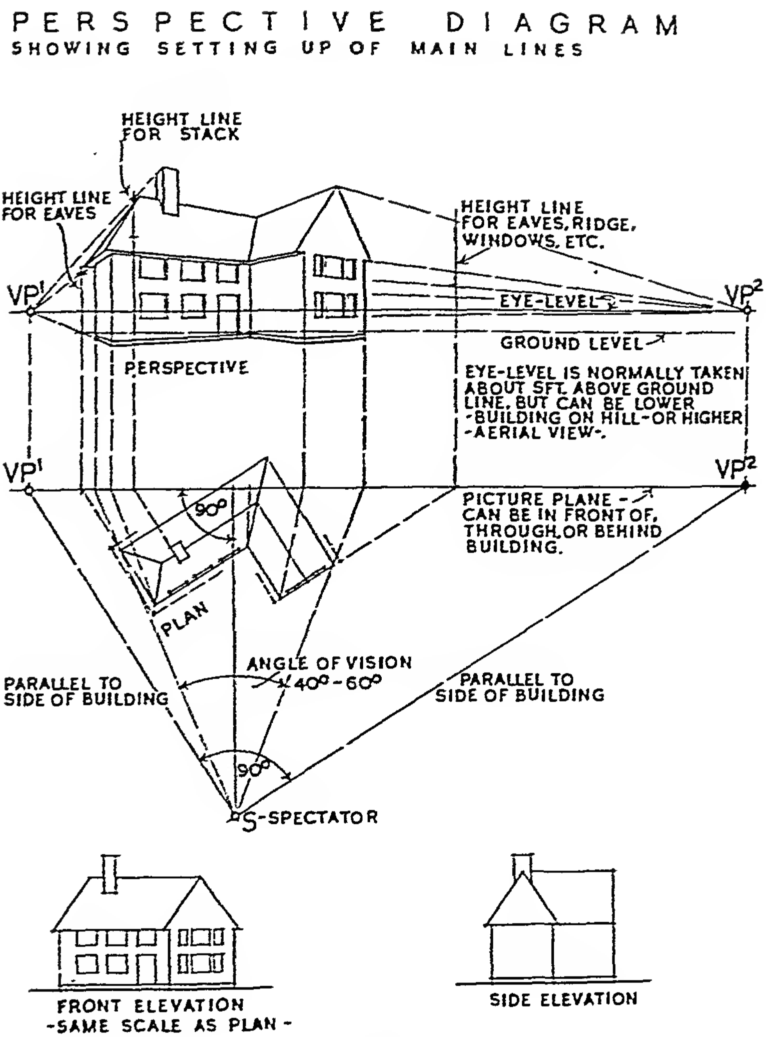

PERSPECTIVE DIAGRAM
SHOWING SETTING UP OF MAIN LINES
HEIGHT LINE FOR STACK
HEIGHT LINE FOR EAVES
HEIGHT LINE FOR EAVES, RIDGE, WINDOWS, ETC.
VP1
VP2
EYE-LEVEL
GROUND LEVEL
PERSPECTIVE
EYE-LEVEL IS NORMALLY TAKEN ABOUT 5FT. ABOVE GROUND LINE, BUT CAN BE LOWER -BUILDING ON HILL- OR HIGHER -AERIAL VIEW-.
VP1
VP2
90°
PICTURE PLANE - CAN BE IN FRONT OF, THROUGH, OR BEHIND BUILDING.
PLAN
ANGLE OF VISION 40°-60°
PARALLEL TO SIDE OF BUILDING
PARALLEL TO SIDE OF BUILDING
90°
S-SPECTATOR
FRONT ELEVATION -SAME SCALE AS PLAN-
SIDE ELEVATION

Fig. 219

It will be seen that in this particular example only one height line is necessary, as when one side of the block has been drawn in perspective the other can be drawn from it. It will also be seen that if the picture plane had been made to pass through any corner of the block the vertical line representing that corner would have served as the height line. In simple cases, therefore, it is common to make the picture plane pass through the nearest corner of the plan, but this practice has disadvantages when the object is complex in form.

Fig. 219 is a diagram to show the method of perspective projection described above applied to a building. The plan should show all relevant recesses and projections, roof lines, etc., and corresponding elevations to the same scale should be available. The position of the spectator is again determined by the limits of the angle of vision, although the immediate surroundings of the building may be included if desired. A useful guide is to place the spectator from the nearest corner of the building a distance equal to three times the height from ground to eaves. The spectator should not be placed on a line bisecting the nearest angle of the building or the effect in the picture will be unsatisfactory. A good result is usually obtained with a rectangular plan by arranging for the picture plane to be parallel to a diagonal.

CHAPTER XVI

POINTS, LINES AND PLANES

Inclined and Oblique Lines and Planes; Dihedral Angle; Penetration of Planes

Points

A POINT in space can be accurately located by three axes or direction lines as shown in Fig. 220. *A*, *B*, *C* are three co-ordinated planes, i.e. each plane is at right-angles to the others. Assuming the point to be 3 units of measurement from *A*, 4 units from *B*, and 2½ units from *C*, it is located by drawing a perpendicular ordinate from plane *C* (horizontal plane) and horizontal co-ordinates from planes *A* and *B* (vertical planes).

Lines

Lines can be perpendicular, horizontal, inclined or oblique. Fig. 221 shows how lines in various positions appear in plan and elevation.

A — perpendicular and clear of the horizontal and vertical planes.
B — perpendicular to vertical plane, clear of horizontal and vertical planes.
C — horizontal and clear of the V.P. and H.P.; parallel to H.P.
D — making full contact with V.P. and H.P.
E — inclined to and clear of the V.P.; parallel to and clear of the H.P.
F — parallel to and clear of the V.P.; inclined to and clear of the H.P.
G — resting on the H.P. and inclined clear of the V.P.

As an example of the drawing of such a line, assume in case *E* that the line is 3 units of measurement long and is inclined at an angle of 30 degrees to the V.P., and is 2 units clear of the H.P. The line is first drawn on the H.P. and perpendicular ordinates are projected up 2 units beyond the *XY* line to give the position of the line in elevation. The true length of an inclined line must always be drawn first in plan or elevation as the case may be.

Fig. 222 shows oblique lines in various positions:

A — inclined to both planes; one end making contact with *XY* line.
B — inclined to both planes; perpendicular to *XY* line.
C — inclined to both planes; not making contact with either plane.
D — inclined to both planes; one end in contact with H.P., the other end in contact with V.P.

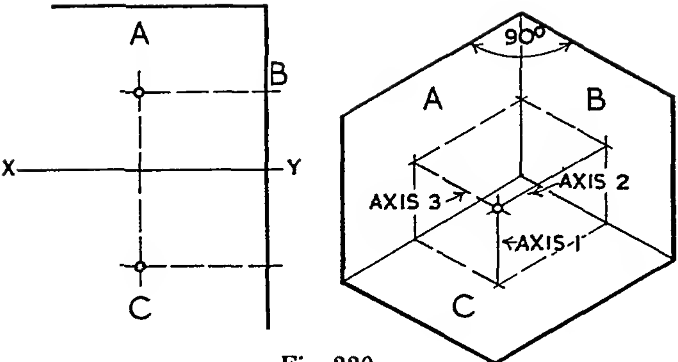


Fig. 220

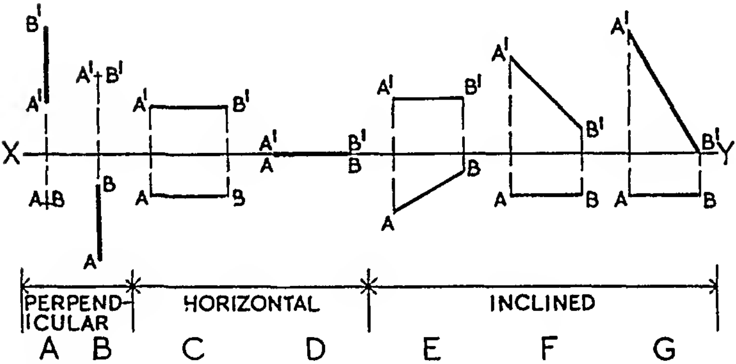


Fig. 221

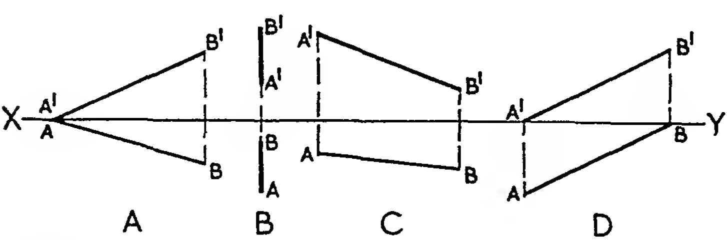


Fig. 222

It will be seen that the true lengths of the lines cannot be shown in plan or elevation, and to deal with the various applications of oblique lines in solid and practical geometry the true lengths must be found. There are alternative methods of doing this.

Figs. 223 and 224 show two pictorial views of the H.P. and V.P. In each drawing an oblique line AB is shown on the surface of a half cone. The vertex of the cone in each case represents one end of the line, the other end of which is at the base. Then, by rotating the line about the surface of the cone its true length can be found on the H.P. or V.P.

Fig. 225 in plan and elevation relates to the pictorial view of Fig. 223. The line AB is drawn at an angle of 30 degrees to the V.P. and 45 degrees to the H.P.; one end contacting both planes. The true length of the line is found by drawing an arc from A with centre B to contact the XY line, from which point a perpendicular is drawn to cut a horizontal line from A^1. By drawing from the point of intersection to B the true length of the line is found. Similarly, by drawing an arc from A^1, as shown.

Another method is also shown in Fig. 225: an auxiliary elevation is produced on AB by drawing a line at right-angles from A and by marking on it the vertical distance of A^1 from the XY line, giving point A^2, which joined to B gives the true length.

Fig. 226 relates to the pictorial view of Fig. 224, the oblique line being drawn at an angle of 60 degrees to the V.P. and 60 degrees to the H.P., with one end contacting the V.P. The true length of the line is found by taking B as centre and with BA as radius drawing an arc to contact the XY line at A^2, which joined to B^1 gives the true length. Similarly, by taking A^1 as centre and A^1B^1 as radius, as shown. The true length of the line can also be obtained by means of an auxiliary plan or elevation as previously described.

Fig. 227 is a further illustration of the methods used to find the true length of an oblique line

Fig. 223

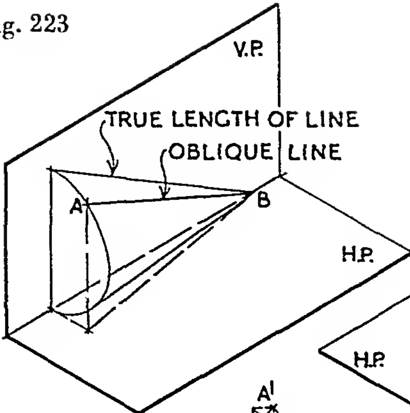
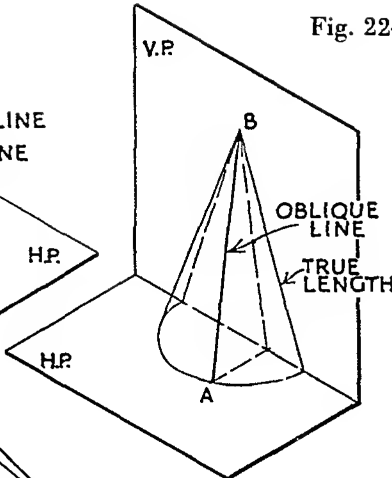


Fig. 224



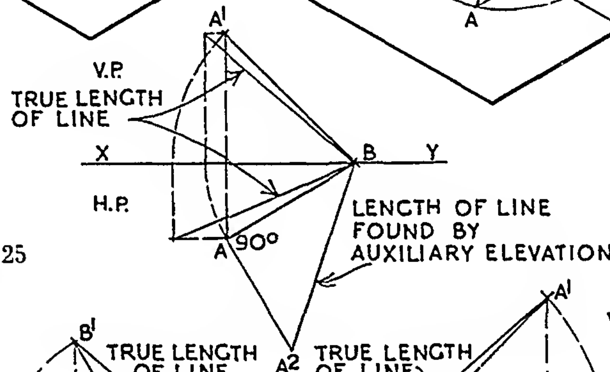
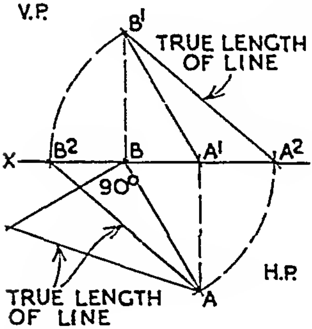


Fig. 225



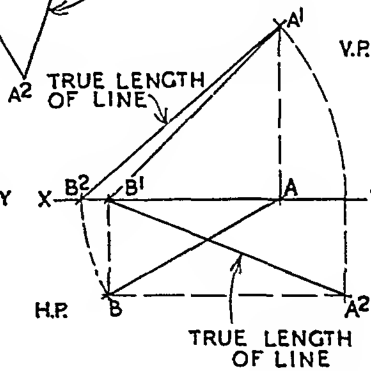


Fig. 226

Fig. 227

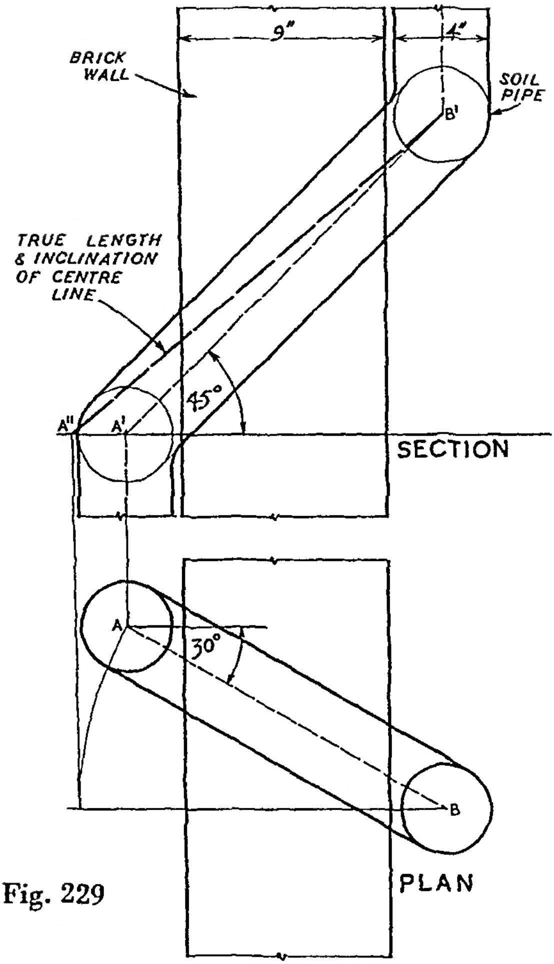


Fig. 229

To obtain the true length and inclination of part of a 4" lead soil pipe passing obliquely through a 9" brick wall (Fig. 229):

This is solved by working from the centre line of the pipe, thus *AB* is the H.T. and *A'B'* is the V.T. (See Oblique Planes, p. 145, for explanation of H.T. and V.T.) With centre *B* and radius *BA* an arc is described to cut a horizontal line drawn through *B*; project the point of intersection to a horizontal line drawn through *A'* to find point *A''*, which, joined to *B'*, gives the true length and inclination of the centre line.

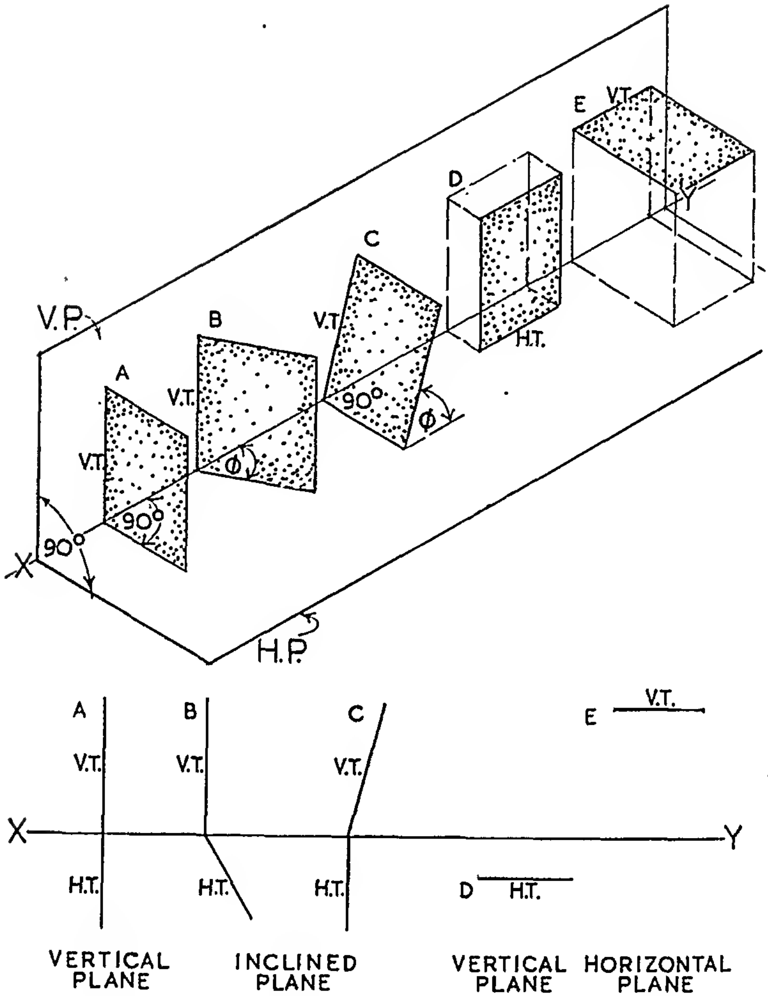


Fig. 230

Planes

There are four classes of planes: (1) Horizontal, (2) Vertical, (3) Inclined, (4) Oblique.

Fig. 230 shows pictorially examples of 1, 2 and 3, and these are projected according to the same principle described for the projection of lines, and it is unnecessary therefore to repeat the explanation.

Oblique Planes

Fig. 231 shows pictorially examples of oblique planes. It will be noticed that in Figs 230–234 the letters V.T. and H.T. are used to denote vertical trace and horizontal trace respectively. A trace is a line bounding a plane. Thus such a line on the vertical plane is the vertical trace, and a line on the horizontal plane is the horizontal trace.

It should be noted that the horizontal and vertical traces of a plane intersect always on the ground line, XY. Where one trace is parallel to the XY line, so also is the other, and the intersection of the three lines is at an infinite distance away, in the direction of the ground line.

To distinguish an oblique plane from one which is perpendicular to one or both of the planes of projection, note that neither trace is an edge or profile view of the plane.

In Figs 233 and 234 pictorial views of co-ordinate planes are shown with half right cones, against which rest secondary planes. It will be seen that in each case the traces are tangent lines to the base and vertex of the cone.

In Fig. 235, the traces, AB and AC^1, of an oblique plane are shown. The H.T. (AB) is at an angle of 30 degrees to the V.P., and the V.T. (AC^1) is at an angle of 60 degrees to the H.P. The true inclination of the plane is found by taking C as centre and with radius tangent to the H.T., describing an arc to contact the XY line at X^1. By joining X^1 to C^1 the true inclination is found.

To find the development of the plane an arc, with centre A and radius AC^1, is described to contact the projection of CX at C^2. C^2AB is the developed surface of the plane.

Fig. 236 shows the traces, AB^1 and AC, of another oblique plane. The H.T. is at an angle of 60 degrees to the V.P. and the V.T. is at an angle of 45 degrees to the H.P. To find the true inclination of the plane and its development the procedure is similar to that described above. With centre C^1 and radius tangent to the V.T. an arc is described to contact the XY line at X^1. By joining X^1 to C the true inclination is found. The development is found by taking centre A and with radius AC describing an arc to contact the projection of C^1X at C^2. C^2AB^1 is the developed surface.

Fig. 237 shows the traces of another oblique plane with the true inclination and the developed surface found as before. It is suggested that the student copies this example on stiff paper or cardboard so that it can be cut out and folded. If the V.P. is bent up at right-angles to the H.P., and the quadrangle A^1B^3BC is folded so that B^3 coincides with B^1, then the developed surface A^1B^3C can be made to correspond with the traces. Similarly,

Fig. 231

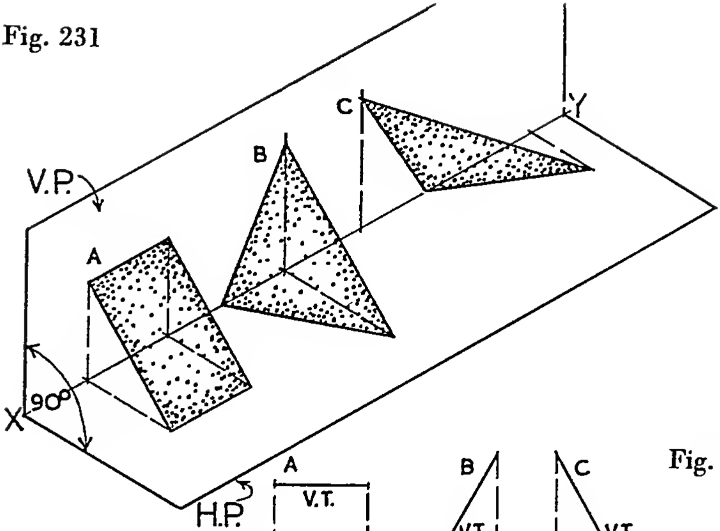
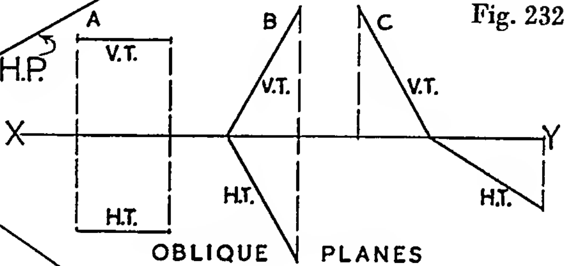
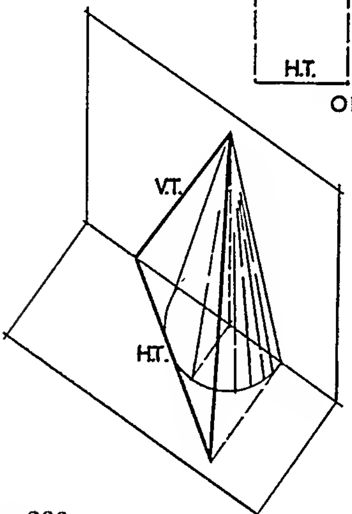


Fig. 232





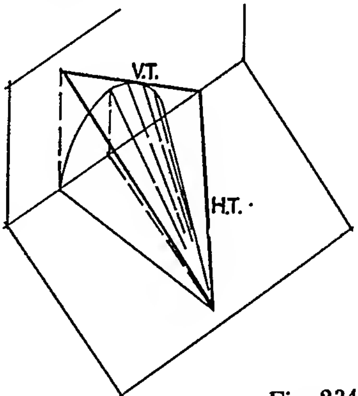
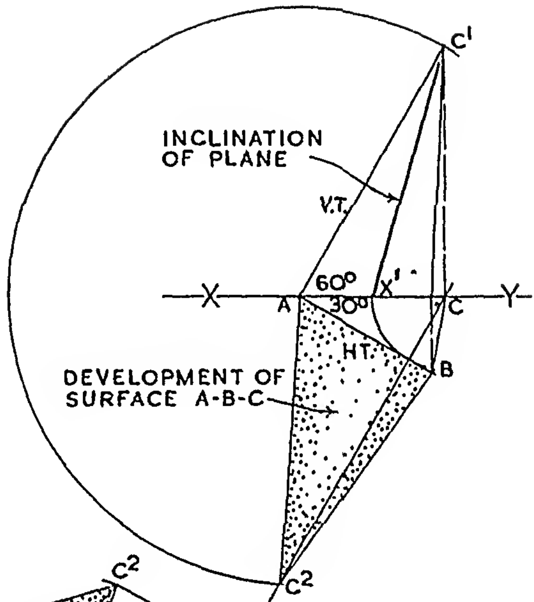


Fig. 233

Fig. 234

Fig. 235



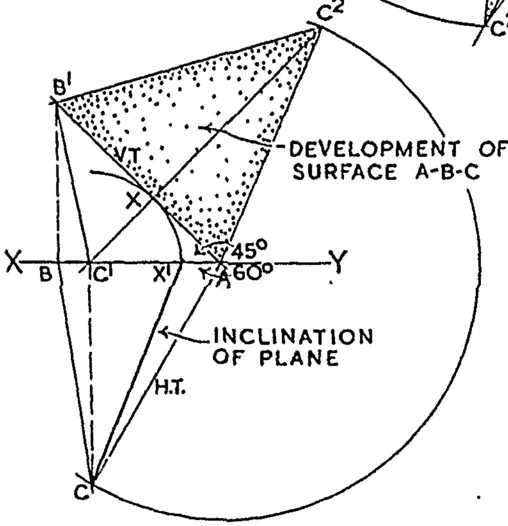
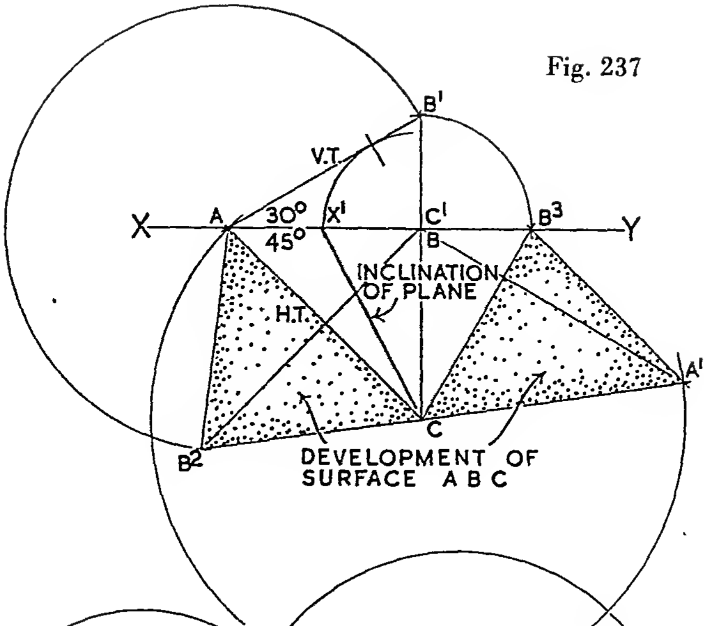


Fig. 236

Fig. 237



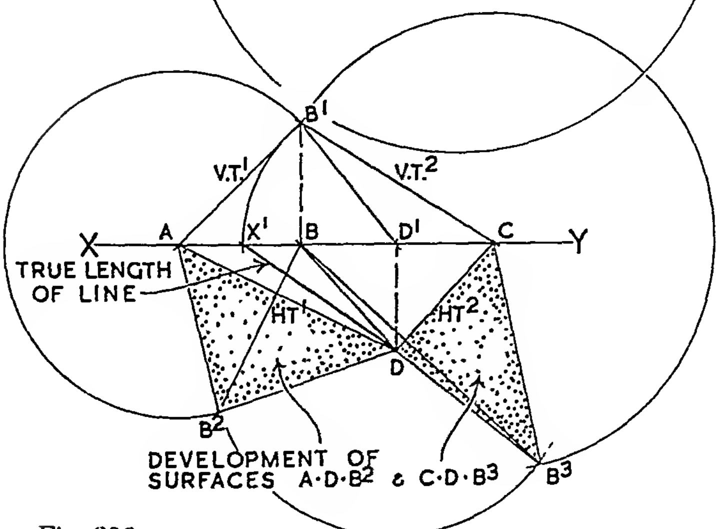


Fig. 238

the developed surface AB^2C can be folded up to cover exactly A^1B^3C.

Models can be made of the previous examples in the same way.

Fig. 238 shows two oblique planes making contact. There are, therefore, two vertical traces (AB^1 and B^1C) and two horizontal traces (AD and DC). To find the line at the junction of the planes, its true length and the development of the two surfaces, the method is as follows: The traces are drawn and indexed. Perpendicular lines are drawn from B^1 and D to the XY line; and lines are drawn from B^1 to D^1, the perpendicular of D, and from D to B, the perpendicular of B^1, thus giving the required line on both the H.P. and the V.P. at the junction of the planes. Then, with centre D^1 and radius D^1B^1 an arc is described to the XY line. From the point of contact, X^1, to D is the true length of the line at the junction of the planes. Finally, a line at right-angles to AD is drawn from B and with centre A and radius AB^1 an arc is described to cut it at B^2. ADB^2 so obtained is the developed surface of one plane, and the developed surface of the other, DCB^3, is found in a similar manner; an arc, radius CB^1, with centre C is described to contact a line from B at right-angles to CD.

Fig. 239 shows a pictorial view of the corner of a hipped roof. It will be seen that the hip of the roof is a line at the junction of two oblique planes, the horizontal traces of which are the wall-plates. Fig. 240 shows the same roof lines in plan and elevation. H.T.^{1} and H.T.^{2} are at right-angles to one another, and V.T. is at an angle of 45 degrees to the H.P. By the application of the foregoing principle, the true length of the hip is found at C^1B^1, and also at C^2B by drawing a line at right-angles to CB from C and contacting it by an arc drawn from C^1 with centre C. This latter operation produces a new elevation. The developed surfaces are found by the method explained for Fig. 235.

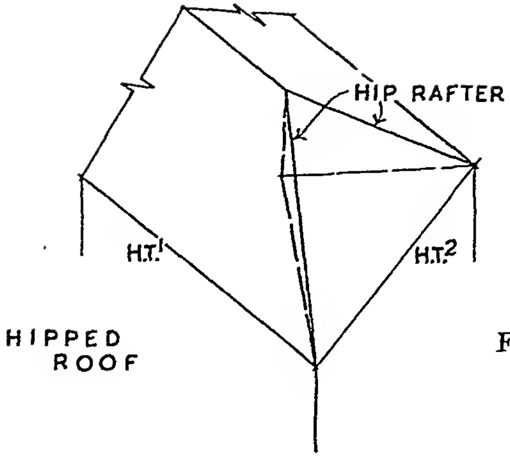


Fig. 239

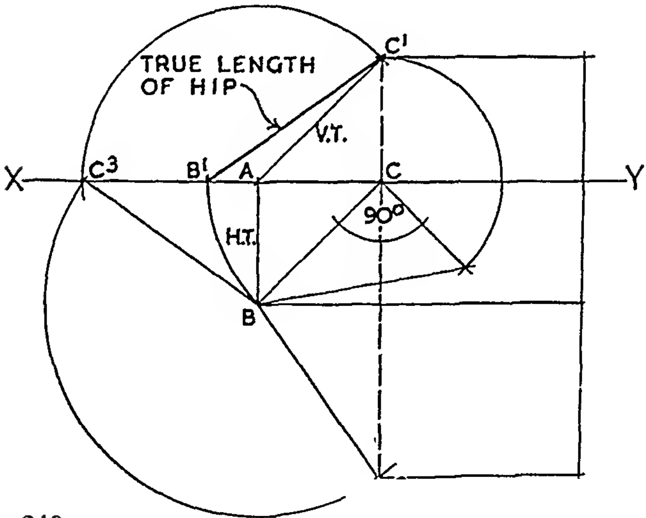


Fig. 240

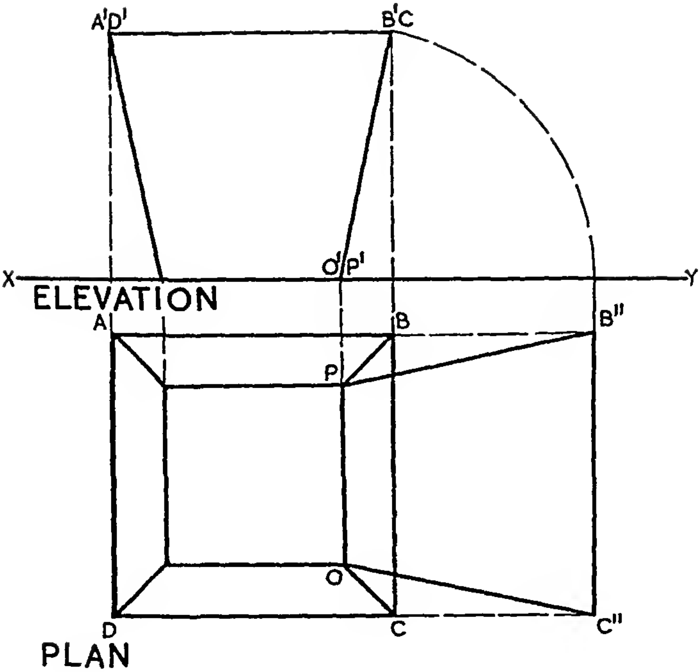


Fig. 241

To find the true shape of the sides of a square wood or metal hopper (Fig. 241):

Draw the plan and elevation of the hopper to a suitable scale. With centre $O'P'$ and radius equal to $O'P'B'C'$ describe an arc to the H.P. and project to the plan to cut AB and DC produced in B'' and C''. Join P to B'' and O to C'', then $PB''C''O$ is the required shape.

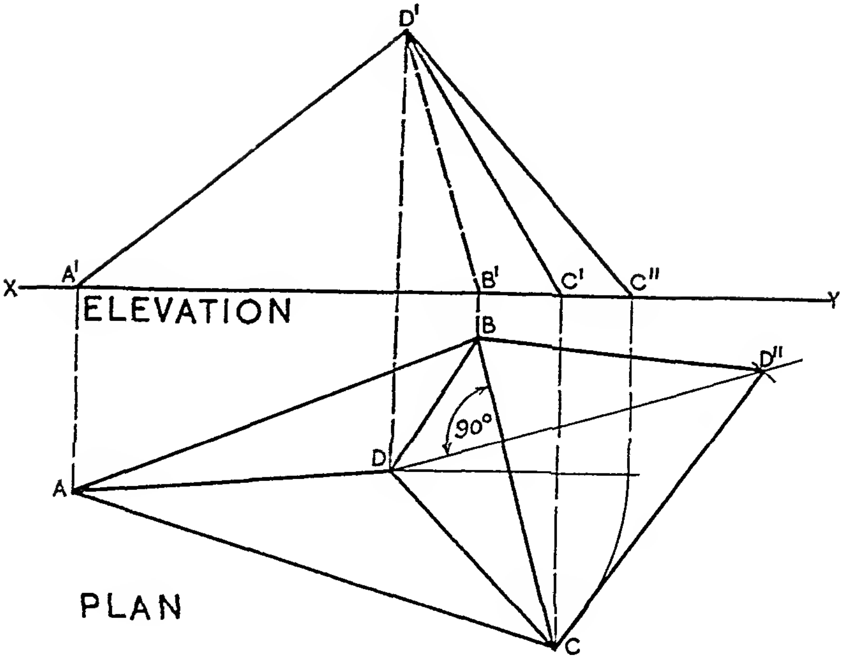


Fig. 242

To find the true shape of the side BCD, and the length of the hip rafter, DC, of a three-sided pyramidal roof (Fig. 242):

Draw the plan and elevation to a suitable scale. With centre D and radius DC describe an arc to cut a horizontal line drawn through D. Project from the point so obtained to the X–Y line to find C'' and join C'' to D'. $C''D'$ is the required true length of the hip rafter (to scale, of course).

From D draw a line perpendicular to BC, and with radius $C''D'$ and centre C describe an arc to cut this line at D''. Join B and C to D'', then $BD''C$ is the required shape of side BCD.

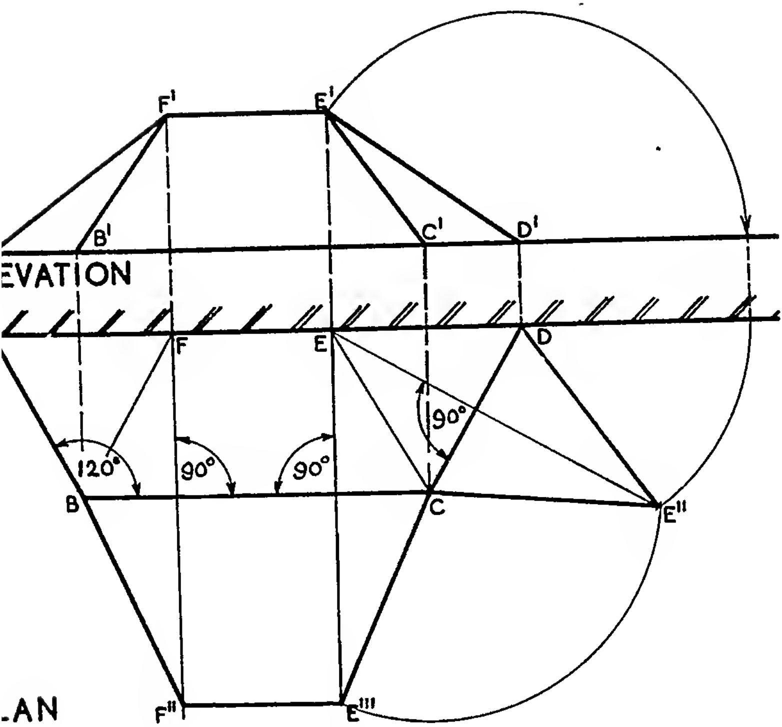


Fig. 243

To find the true shape of the surfaces of the roof of a bay window (Fig. 243):

With centre D' and radius $D'E'$ describe an arc to cut $A'D'$ produced, and project to AD produced on plan. With centre D continue the projection as an arc to cut a line drawn from E perpendicular to CD at E''. $DE''C$ is the true shape of surface DEC and, of course, of AFB.

With centre C and radius CE'' describe an arc to cut a line drawn from E perpendicular to BC at E'''. From E''' draw a line parallel to CB to cut a line drawn from F perpendicular to BC at F''. $CE'''F''B$ is the true shape of surface $CEFB$.

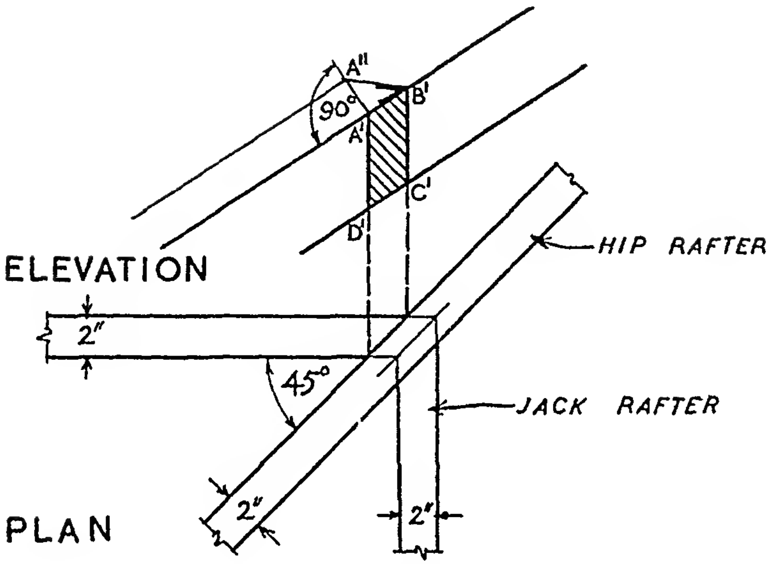


Fig. 244

To find the oblique angle of intersection between the jack rafter and hip rafter of a pitched roof. (This intersection, which occurs at the top of the jack rafter, is known as a cross-cut bevel.) (Fig. 244):

By projecting from the plan $A'B'C'D'$, the cut of the jack rafter, can be found on the elevation of the hip rafter.

At A' draw a perpendicular to $A'B'$ to A'' so that $A'A''$ equals 2", the thickness of the jack rafter. Join A'' to B'. $A''B'A'$ is the required angle.

The Dihedral Angle is the true angle between two intersecting planes, its inclination being perpendicular to the line connecting the planes.

The method of finding the dihedral angle of two contacting planes is shown in Figs. 245 and 246, where the traces of the planes are at angle of 45 degrees and 60 degrees to both the V.P. and H.P. A new elevation of the line joining the planes is first found. Then the line ST is drawn at right-angles to B^3D^1, and a line from S is drawn at right-angles to X^1Y^1 to H.T.[1] and H.T.[2] to contact at S^1 and S^2 respectively. With S as centre and ST

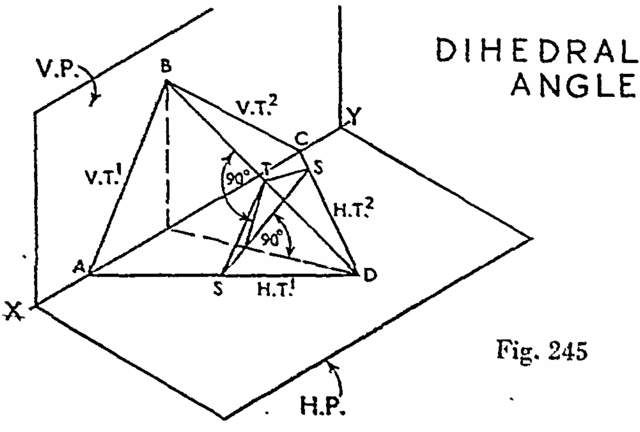


Fig. 245

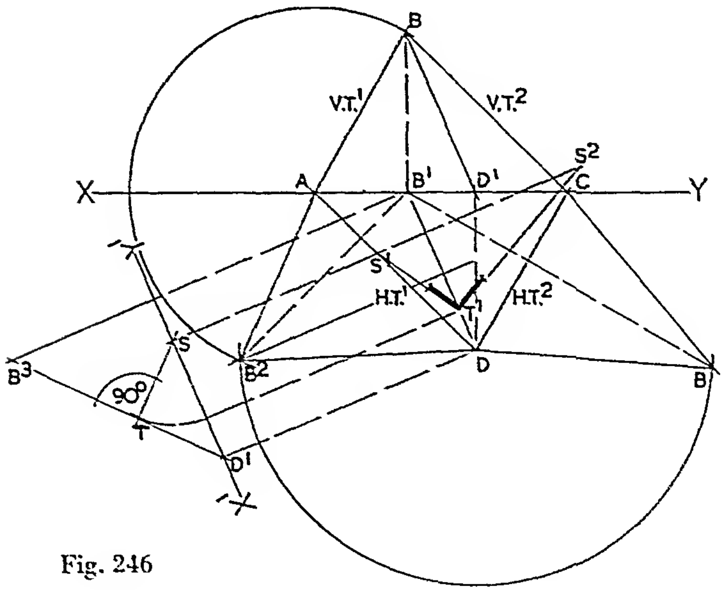


Fig. 246

as radius an arc is described to X^1Y^1 and then continued at right-angles to X^1Y^1 to contact B^1D at T^1. The dihedral angle is found by joining S^1T^1 and T^1S^2. The development of the surfaces of the planes is also shown in the drawing.

Fig. 247 shows another method of obtaining the dihedral angle. The traces of the planes are drawn and the line at the junction of them obtained. The true length of this line, B^1D^1, is found as seen in elevation. With a point T as centre and radius TS tangent to B^1D^1 an arc is described to the XY line and continued as an arc struck with B as centre to cut BD at S^1. Also with B as centre an arc is described from T to cut BD, at which point a line at right-angles to BD is drawn to contact AD at T^1 and CD at T^2. By joining $T^1S^1T^2$ the dihedral angle is found.

Fig. 248 shows in plan (H.P.) and elevation (V.P.) a rectangular chimney stack penetrating a pitched roof surface (oblique plane). By the application of the methods described the developed surface of the roof is found, and the true shape of the hole caused by the penetration of the stack is found with the help of a new elevation.

DIHEDRAL
ANGLE

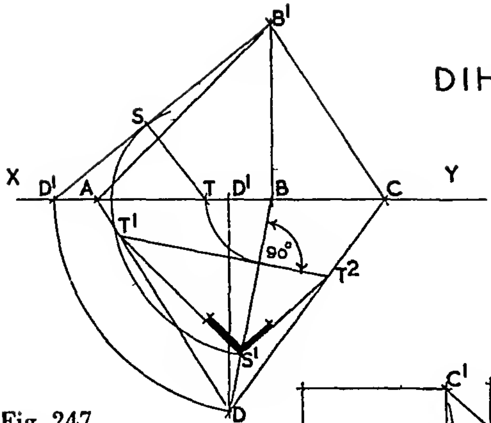


Fig. 247

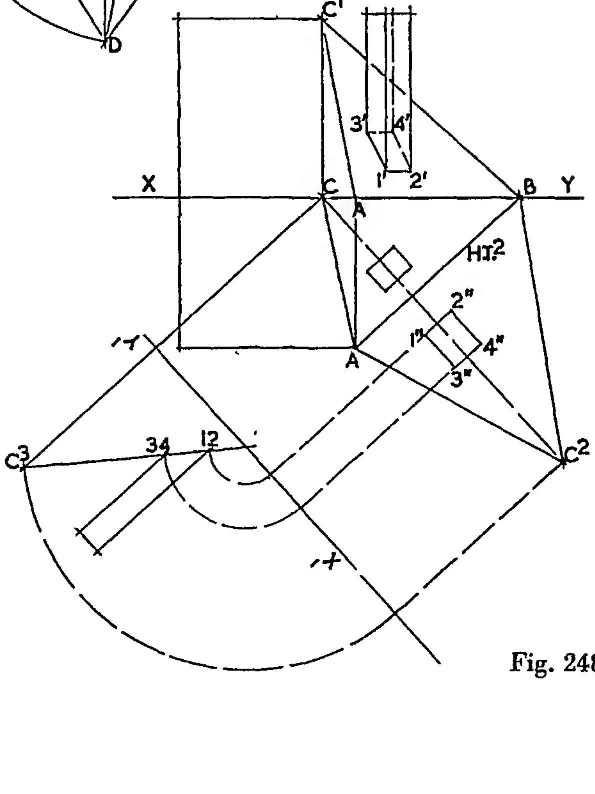


Fig. 248

CHAPTER XVII

INCLINED PROJECTIONS OF SOLIDS

Plotting Auxiliary Views of Solids

A solid figure may be placed in various positions in relation to the horizontal and vertical planes (H.P. and V.P.). If an object or building is rectangular on plan and is placed with its front side parallel to the vertical plane, only that side will be seen in elevation (orthographic projection). But if the plan or elevation is moved so that one or other is inclined to the horizontal plane or vertical plane then the projection is an inclined projection.

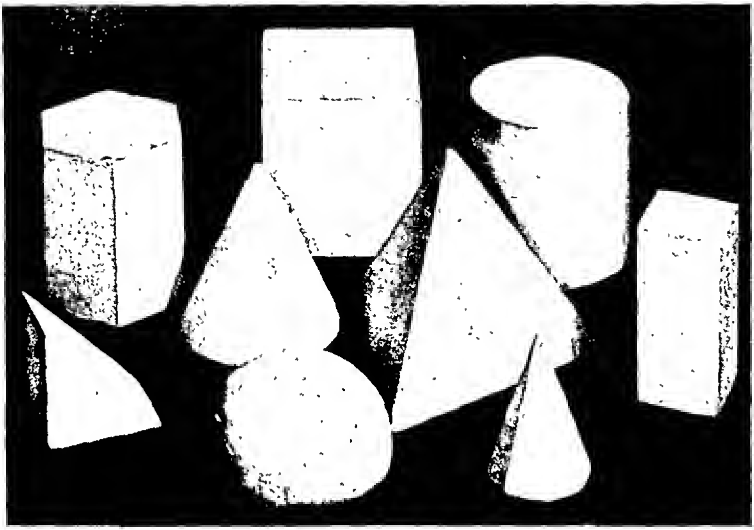
Inclined Projection

Assuming the object or building to be drawn with the side of the plan inclined at an angle of 30 degrees to the V.P., then the front view would result in two distorted elevations, although the height would be unaffected.

Figs. 249 and 250 show pictorial views of a rectangular block (which may represent the basic form of a brick or a building) placed in inclined positions. In Fig. 249 the block is placed flat on the H.P. and with its side at an angle of 30 degrees to the vertical plane. In Fig. 250 its edge rests on the horizontal plane, its long side parallel to the vertical plane but tilted at an angle of 30 degrees to the horizontal plane. In both cases the block is in an inclined position.

Fig. 251 shows how the block in Fig. 249 is drawn in plan and elevation. The plan, *ABCD*, is first drawn to scale so that *AB* is inclined at an angle of 30 degrees to the V.P. An auxiliary elevation is then drawn. From the plan perpendicular ordinates *A*, *B*, *C*, *D* are projected to contact corresponding horizontal co-ordinates projected from the auxiliary elevation. The elevation is thus obtained.

Fig. 252 shows how Fig. 250 is similarly drawn in plan and elevation.



GEOMETRICAL SOLIDS

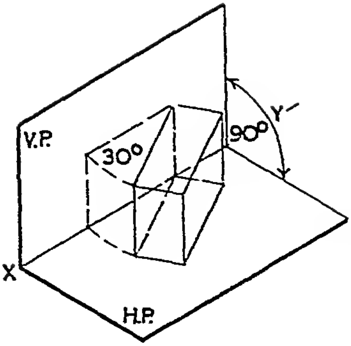


Fig. 249

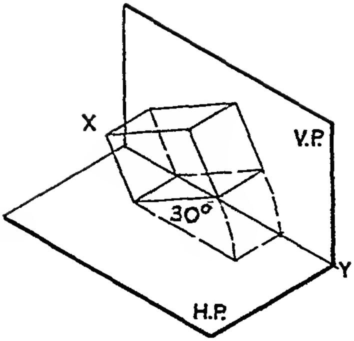


Fig. 250

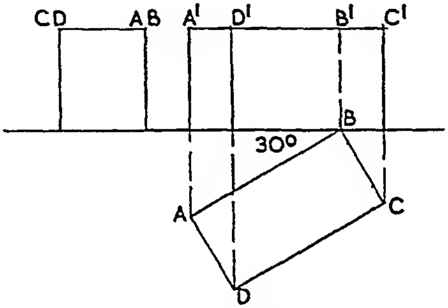


Fig. 251

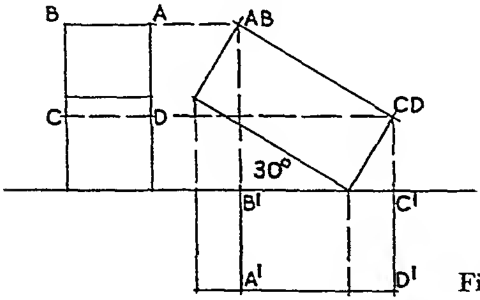


Fig. 252

To draw a triangular prism in plan and elevation when one of its edges is resting on the H.P. and its axis is inclined at 30 degrees to the H.P. and is parallel to the V.P. (Fig. 253):

Draw *ABCDEF*, the auxiliary plan of the prism as though its base were resting wholly on the H.P., and project from *A* and *D* and *B* and *C* to the *XY* line. Draw a line from *A'D'* at 30 degrees to the *XY* line and with centre *A'D'* and radius equal to *AB* describe an arc to cut this line at *B'C'*. From *A'D'* and *B'C'* erect perpendiculars to *A'B'* equal in length to the vertical height of the prism—see end view of prism. *A'D'*, *E'*, *F'*, *B'C'* is the required elevation. To find the required plan, $AB^2C^2DE^2F^2$, project vertical ordinates from the salient points on elevation to contact corresponding co-ordinates from the auxiliary plan.

To draw an octagonal prism in plan and elevation when one of its sides is resting on the H.P. with its axis inclined at 30 degrees to the V.P. and it is clear of the V.P. (Fig. 254):

Draw the horizontal *XY* line and the line of inclination (H.T.) below it, and set out an auxiliary end view, indexed *A* to *H*, as shown. Produce ordinates from the indexed points to obtain the required plan. Set out an auxiliary end view in elevation above the *XY* line indexed to correspond with the plan. Produce horizontal ordinates to contact corresponding co-ordinates projected from the plan to find the salient points of the required elevation.

To draw in plan and elevation an hexagonal pyramid with its base inclined at 30 degrees to the H.P. with its axis parallel to the V.P.; it is clear of the V.P. (Fig. 255):

Draw *ABCDEF*, the auxiliary plan of the pyramid, and produce to the *XY* line and draw the auxiliary elevation, *O* being the vertex. With centre *A'F'* and radius equal to the width of the base describe arcs to the line of inclination (V.T.), to find *B'E'* and *C'D'*. With the same centre and radius *A'F'.O* describe an arc to cut at *O'* a perpendicular to *A'F'*, *C'D'* drawn from *B'E'*. The required elevation can now be drawn. To find the required plan, project vertical ordinates from *A'F'*, *B'E'*, *C'D'* and *O'* to contact corresponding co-ordinates drawn from the auxiliary plan.

To draw in plan and elevation an octagonal pyramid with one of its base edges resting on the H.P., its axis parallel to the H.P. and inclined at 30 degrees to the V.P., and its apex touching the V.P. (Fig. 256):

Draw the *XY* line and the line of inclination and set out auxiliary views of the base in both planes as shown, and index to correspond. Produce from the indexed points as in previous examples and by ordinates and co-ordinates plot the required views.

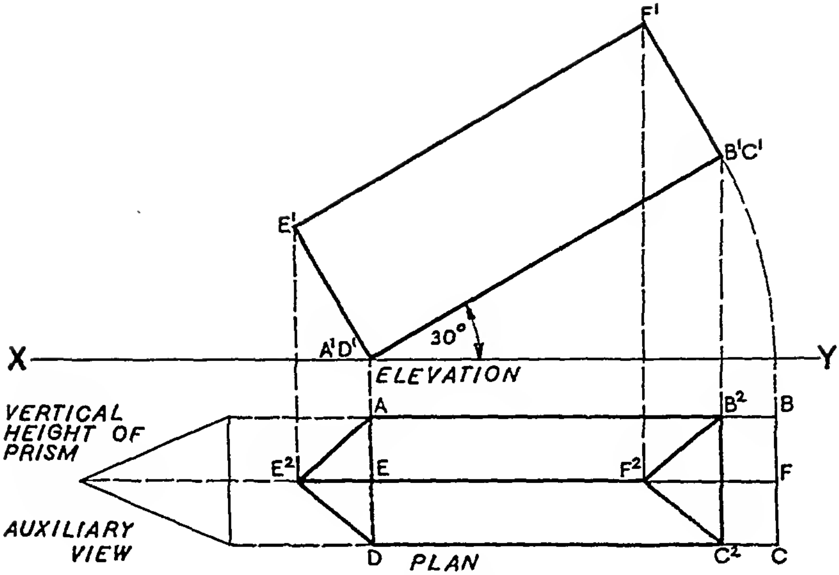


Fig. 253

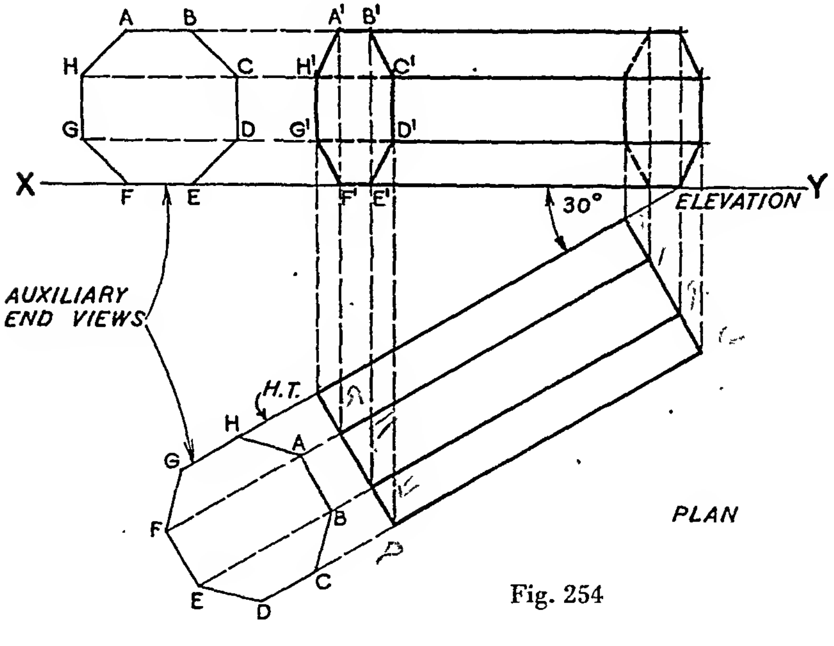


Fig. 254

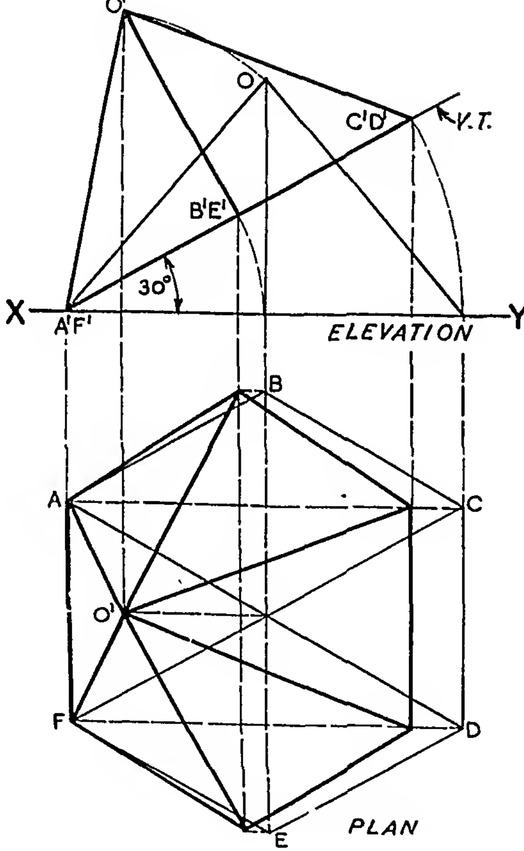


Fig. 255

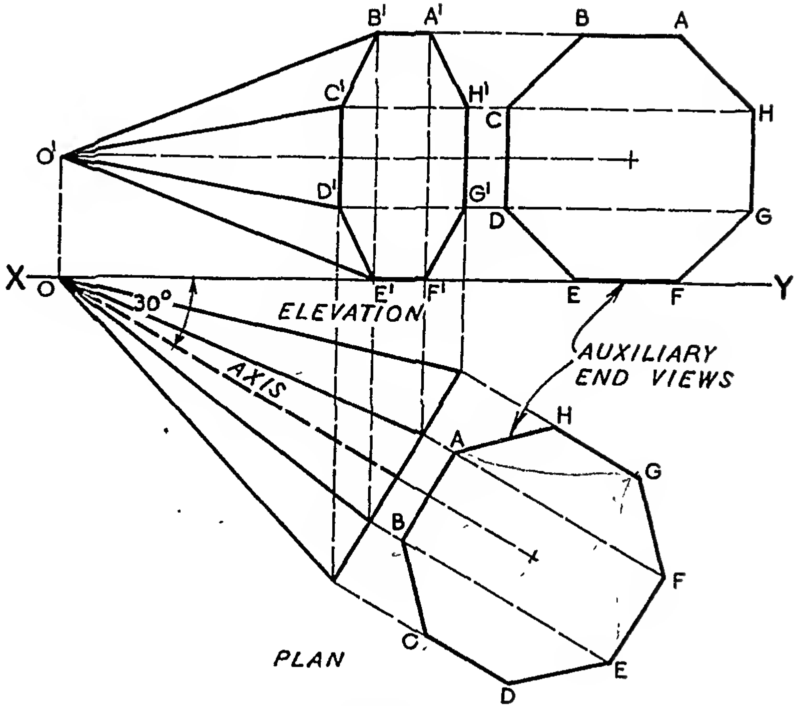


Fig. 256

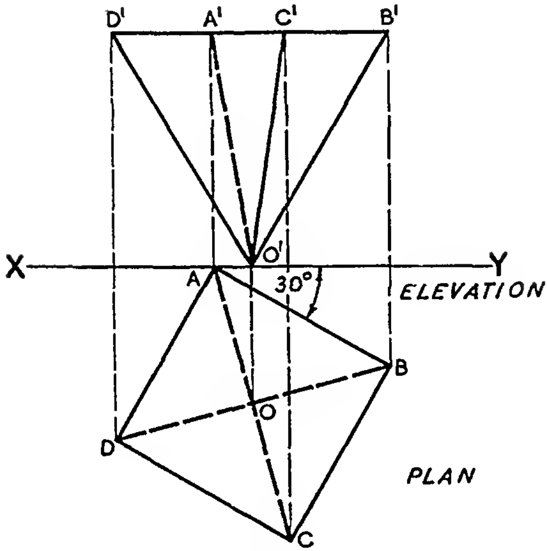


Fig. 257

To draw in plan and elevation a square pyramid resting with its apex on the H.P., its axis perpendicular to the H.P., and one side at 30 *degrees to the V.P.* (Fig. 257):

Draw a plan, $ABCD$, of the pyramid so that one side is at 30 degrees to the horizontal XY line, and produce vertical ordinates to the elevation. Mark off the height of the pyramid along the axis from the XY line and draw a horizontal line to represent the base and to find points A', B', C', and D', which joined to O' complete the required elevation.

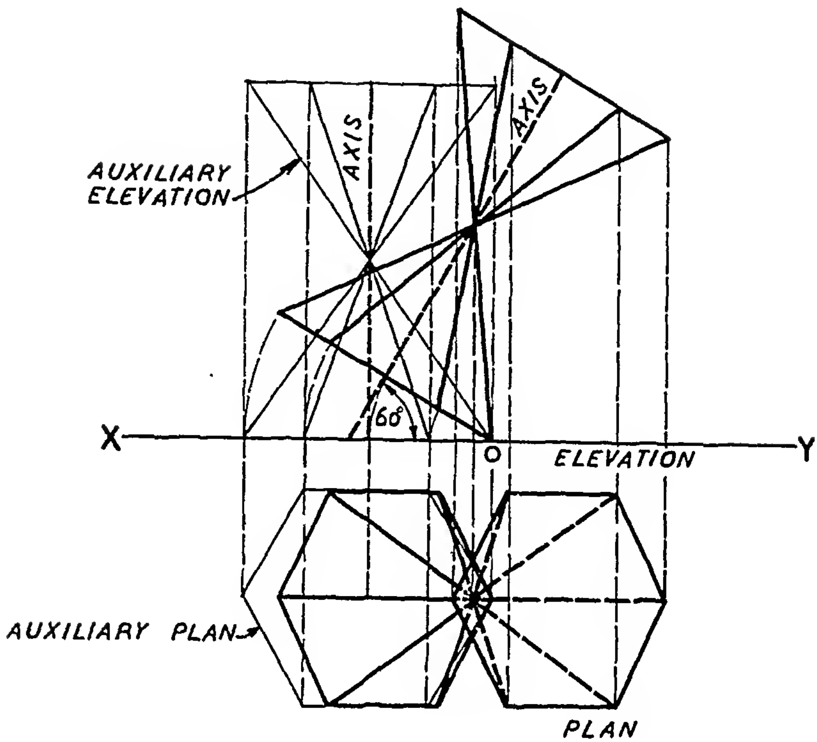


Fig. 258

The vertices of two equal hexagonal pyramids are making contact and the figures have a common axis. To draw these figures with the axis inclined at 60 *degrees to the H.P. and parallel to the V.P.* (Fig. 258):

Draw the auxiliary plan and elevation. With centre *O* describe arcs from the base of the auxiliary elevation to the new base line which is drawn at 30 degrees to the *XY* line. Then, by following the general procedure already described, plot and draw first the required elevation and then, using ordinates and co-ordinates, the required plan.

The following examples are further illustrations of inclined projections obtained by the methods previously described:

Fig. 259 shows a cone with its side resting on the H.P. and its axis parallel to the V.P.

Fig. 260 shows a cylinder with its side inclined at 30 degrees to the H.P. and its axis parallel to the V.P.

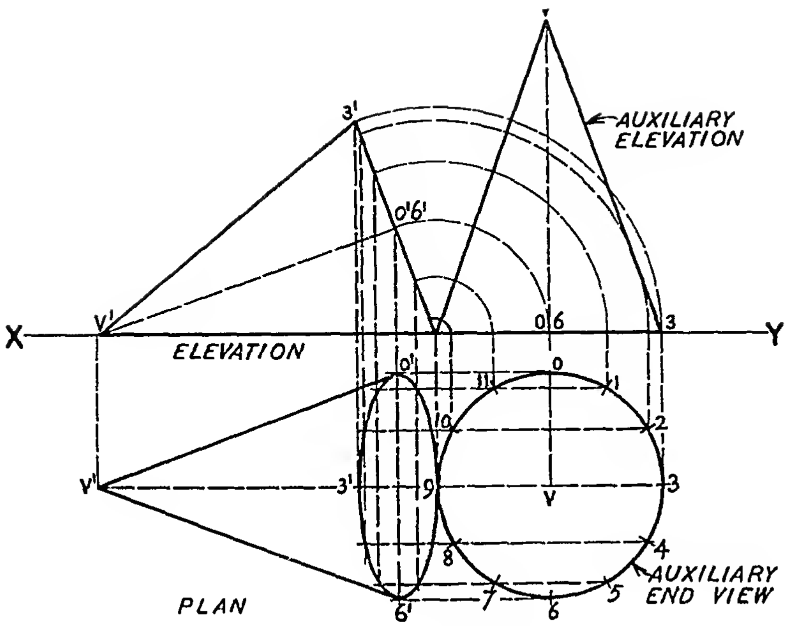


Fig. 259

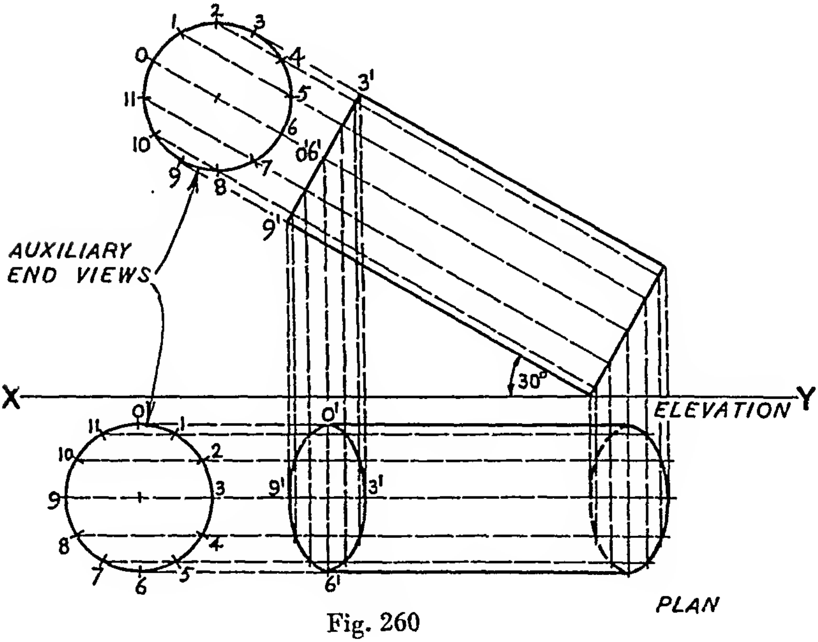


Fig. 260

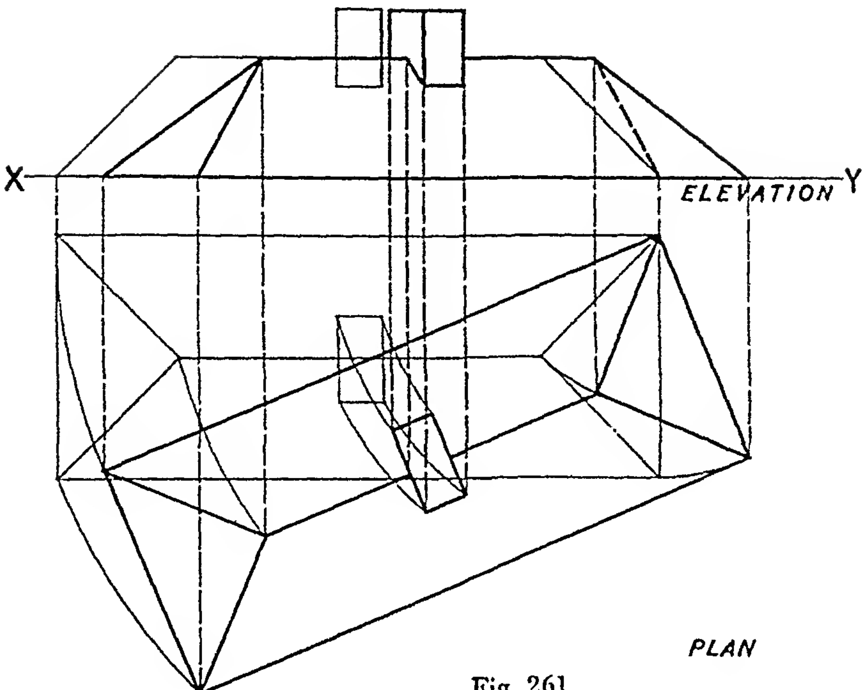


Fig. 261

Fig. 261 shows a simple hipped roof penetrated by a chimney stack, the whole turned at an angle to the V.P.

CHAPTER XVIII

ALTERATION OF GROUND LINE AND SECTIONAL PROJECTION

New Plans, Elevations and Sections of Geometrical Solids and Building Forms, Contouring

New Plans and Elevations

The purpose of projecting new plans and elevations is to obtain views of at least two adjoining faces of the object either in plan or elevation. The following principles should be noted:

An *auxiliary elevation* is a projection on any vertical plane which is not parallel to the principal vertical plane, as shown in Fig. 262, where A^2 is the auxiliary elevation of A on the auxiliary V.P.

An *auxiliary plan* is a projection on any plane which is perpendicular to the V.P. and not parallel to the H.P., as shown in Fig. 263, where A^2 is the auxiliary plan of A on the auxiliary H.P.

Figs 264 and 265 illustrate how the auxiliary planes can be turned into a common plane for two-dimensional representation.

New elevations. If two or more elevations are projected from any one plan the distances of the various elevations from their respective ground lines must be equal.

New plans. If two or more plans are projected from any one elevation, the distances of the various plans from their respective lines must be equal.

Fig. 266 gives a pictorial view of a rectangular solid standing parallel to the V.P. To obtain a new plan to be viewed in the direction of the arrow, the new X–Y line will be inclined at an angle of 45 degrees to the principal H.P.

Fig. 267 gives a pictorial view of a similar solid also standing parallel to the V.P. A new elevation is shown, with its ground line at an angle of 60 degrees to the principal V.P.

Figs. 268 and 269 in both cases show how the above views are seen projected on to common planes.

Fig. 270 gives a pictorial view of a rectangular solid cut by a plane perpendicular to the H.P. and at an angle of 45 degrees to the V.P. The projection of this new elevation as shown is an example of a sectional elevation.

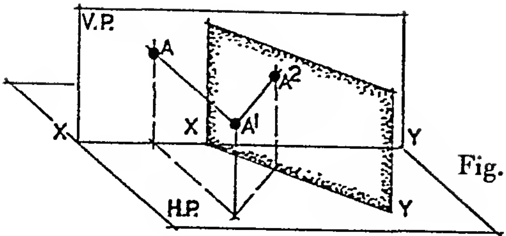


Fig. 262

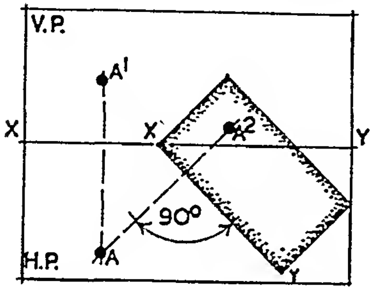


Fig. 263

Fig. 264

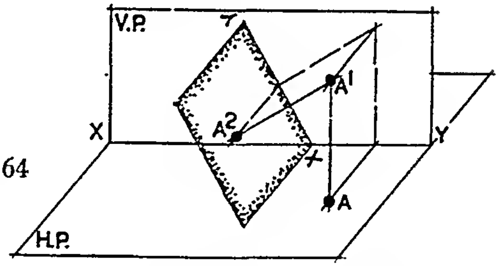


Fig. 265

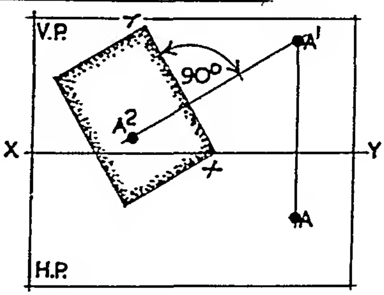
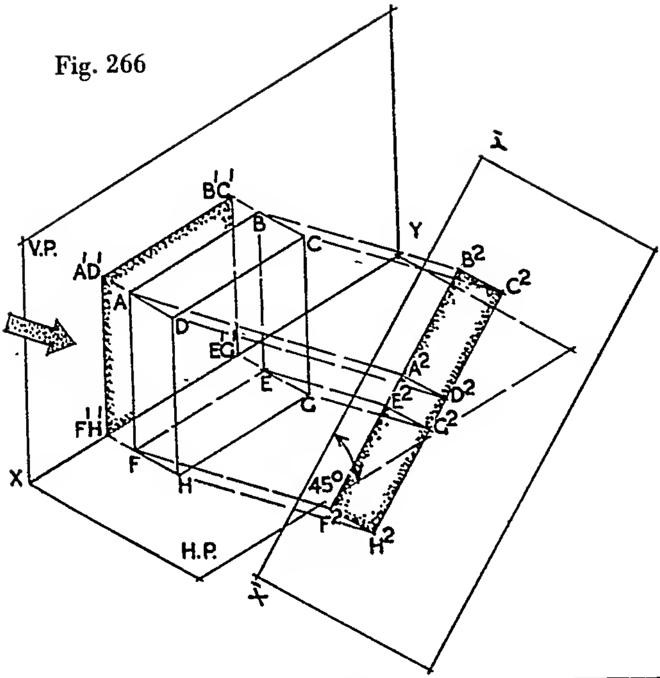


Fig. 266



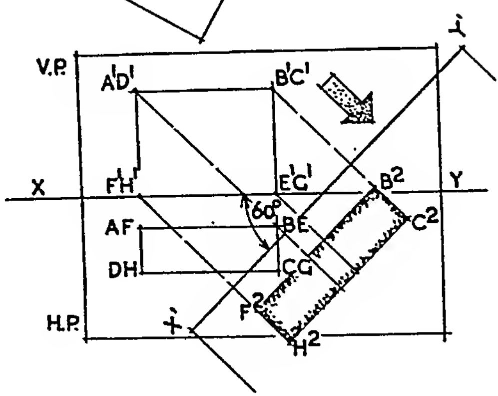
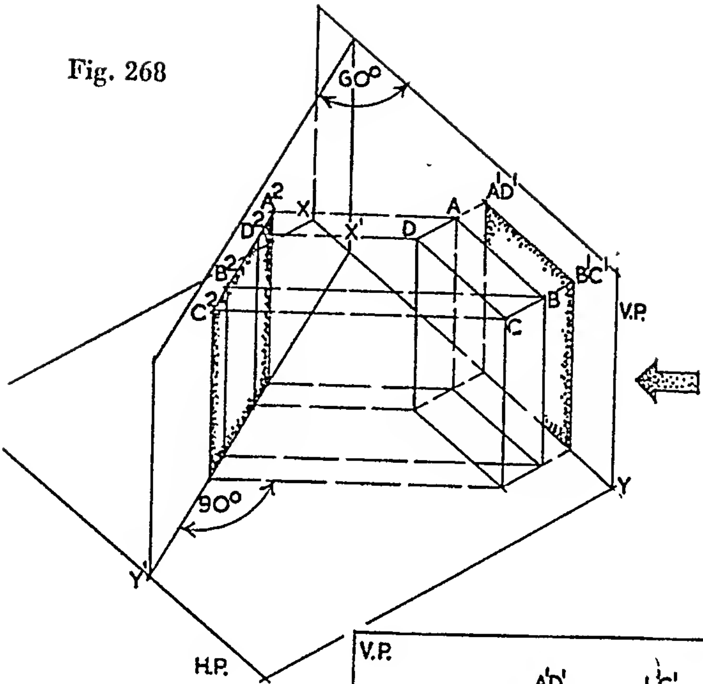


Fig. 267

Fig. 268



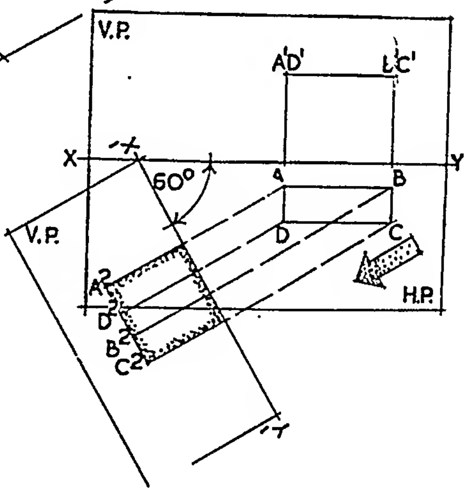


Fig. 269

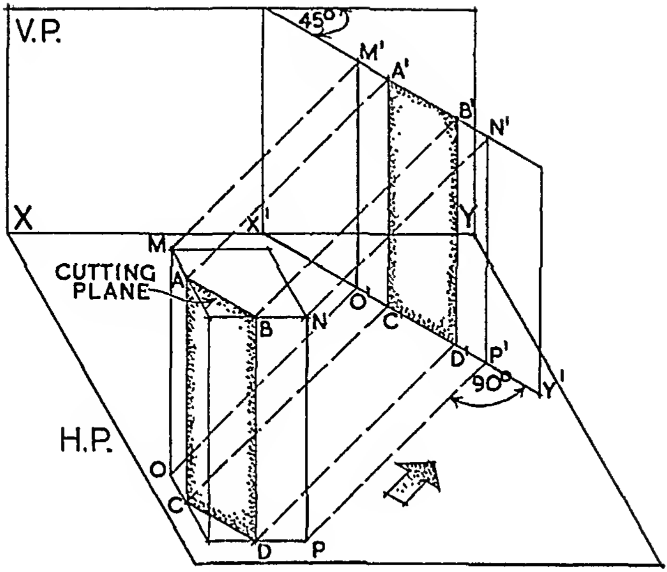
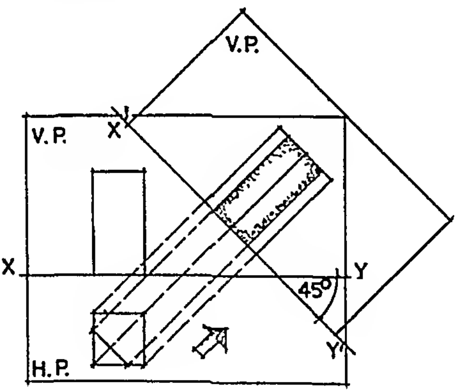

V.P.
45°
M'
A'
B'
N'
X
X'
Y
M
CUTTING
PLANE
A
B
N
O'
C'
D'
P'
90°
Y'
H.P.
O
C
D
P

Fig. 270


V.P.
X'
V.P.
X
Y
45°
Y'
H.P.

Examples

To draw a new elevation of a rectangular solid viewed in the direction of the arrow when the new X–Y line is at 30 *degrees to the original X–Y line* (Fig. 271):

Draw the original or auxiliary plan and elevation of the solid with the base on the H.P. and its side clear of, and parallel to, the V.P. Project ordinates from the indexed points on plan perpendicular to the new *X–Y* line, and plot heights from the original elevation.

To draw a new plan of a rectangular solid viewed in the direction of the arrow when the new X–Y line is at 30 *degrees to the original X–Y line* (Fig. 272):

Draw the auxiliary plan and elevation of the solid with the base on the H.P. and its side clear of, and parallel to, the V.P. Project ordinates from the indexed points on elevation perpendicular to the new *X–Y* line and plot width dimensions from the original plan.

To draw a new plan of a hexagonal pyramid viewed in the direction of the arrow when the new X–Y line is at 45 *degrees to the original X–Y line* (Fig. 273):

Draw the auxiliary plan and elevation of the pyramid with its base resting on the H.P. and one base edge clear of, and parallel to, the V.P. Draw the new *X–Y* line and project ordinates from the indexed elevation perpendicular to it. Plot the required dimensions from the original plan.

In the following examples the procedure is similar to the foregoing.

To draw a new plan of an octagonal prism viewed in the direction of the arrow when the new X–Y line is at 45 *degrees to the original X–Y line* (Fig. 274).

To draw a new plan of a cylinder viewed in the direction of the arrow when the new X–Y line is at an angle of 60 *degrees to the original X–Y line* (Fig. 275).

To draw a new plan of a hexagonal pyramid viewed in the direction of the arrow when the new X–Y line is at an angle of 45 *degrees to the original X–Y line* (Fig. 276).

To draw a true sectional projection of a cutting plane through an equilateral triangular prism standing with its base on the H.P. and one side inclined at 20 *degrees to the V.P. The cutting plane is perpendicular to the H.P. and is inclined at* 30 *degrees to the V.P.* (Fig. 277).

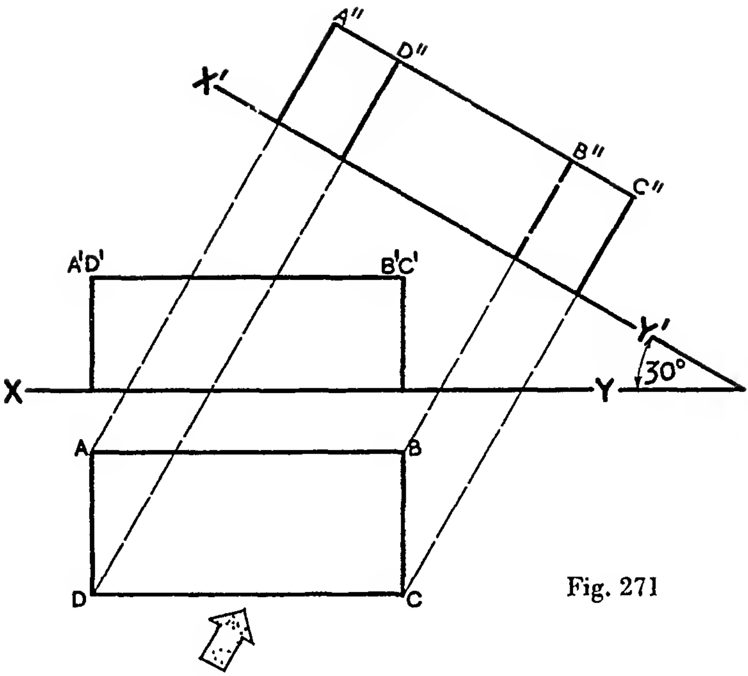


Fig. 271

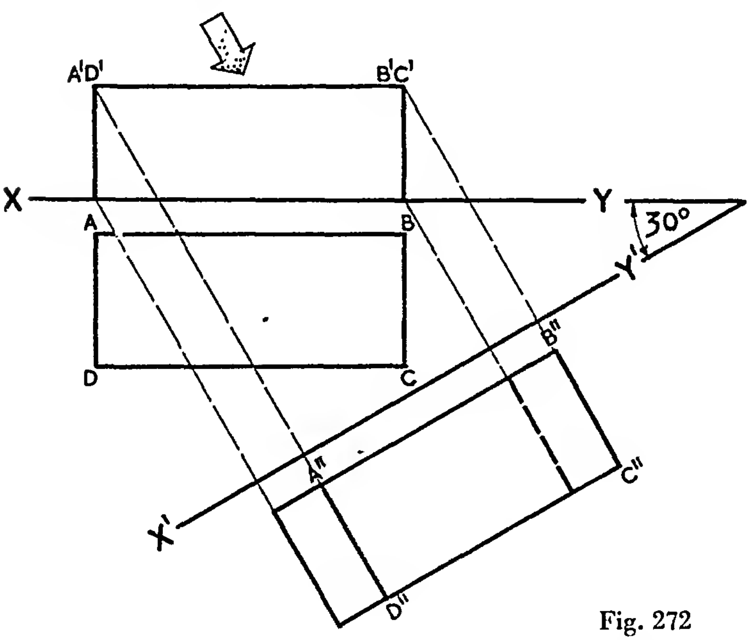


Fig. 272

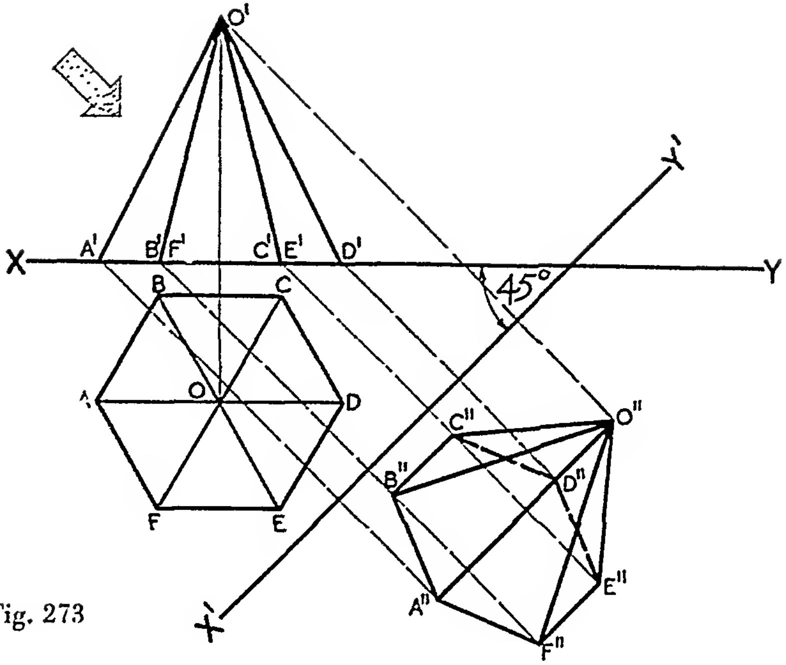


Fig. 273

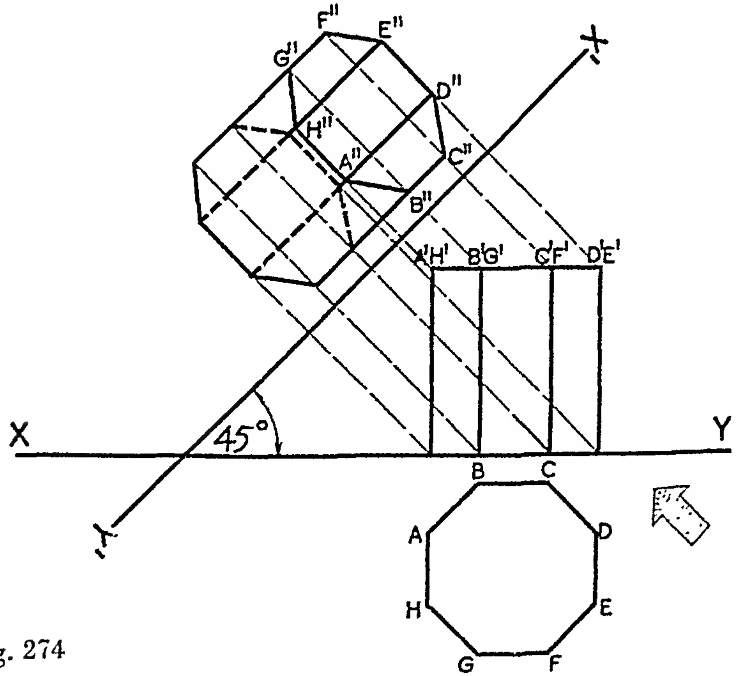


Fig. 274

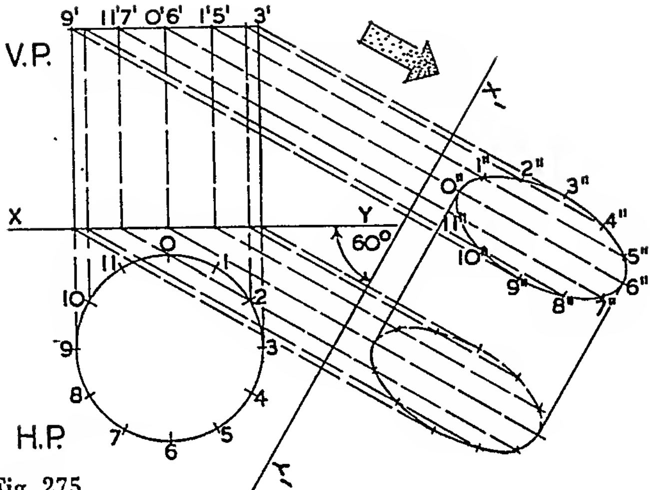


Fig. 275

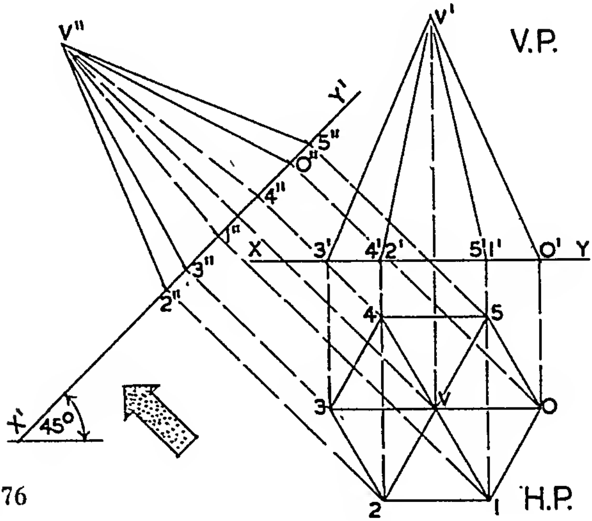


Fig. 276

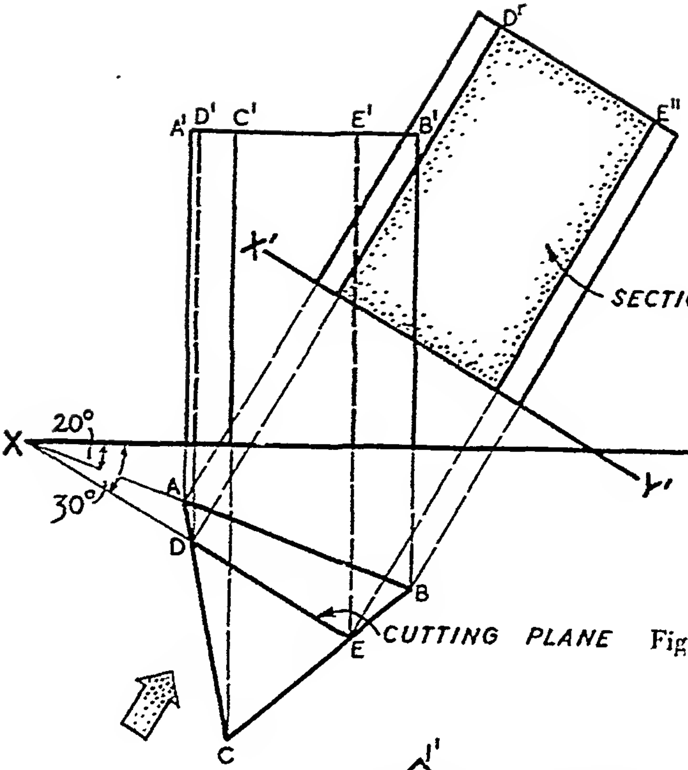


Fig. 277

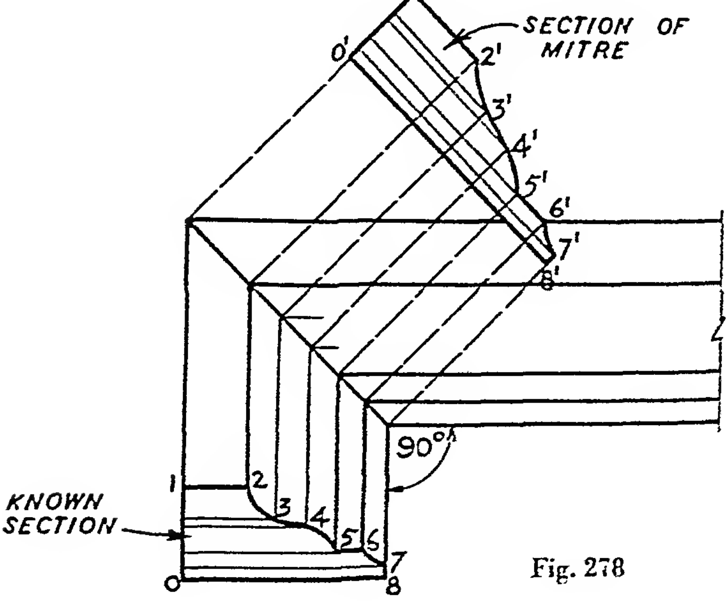


Fig. 278

Fig. 278 shows how the true mitre cut of two equal intersecting mouldings is obtained. It will be seen that an auxiliary section of one of the mouldings is needed.

Fig. 279 shows the plan of a four-hipped roof with all roof *surfaces inclined at 45 degrees to the H.P., with the following* three sectional projections: (*a*) The projection of hip-rafter *BC* giving its true length and inclination. (*b*) The projection of hip-rafter *AB* giving its true length and inclination. (*c*) The projection through centre of roof as indicated by arrows, i.e. the longitudinal section.

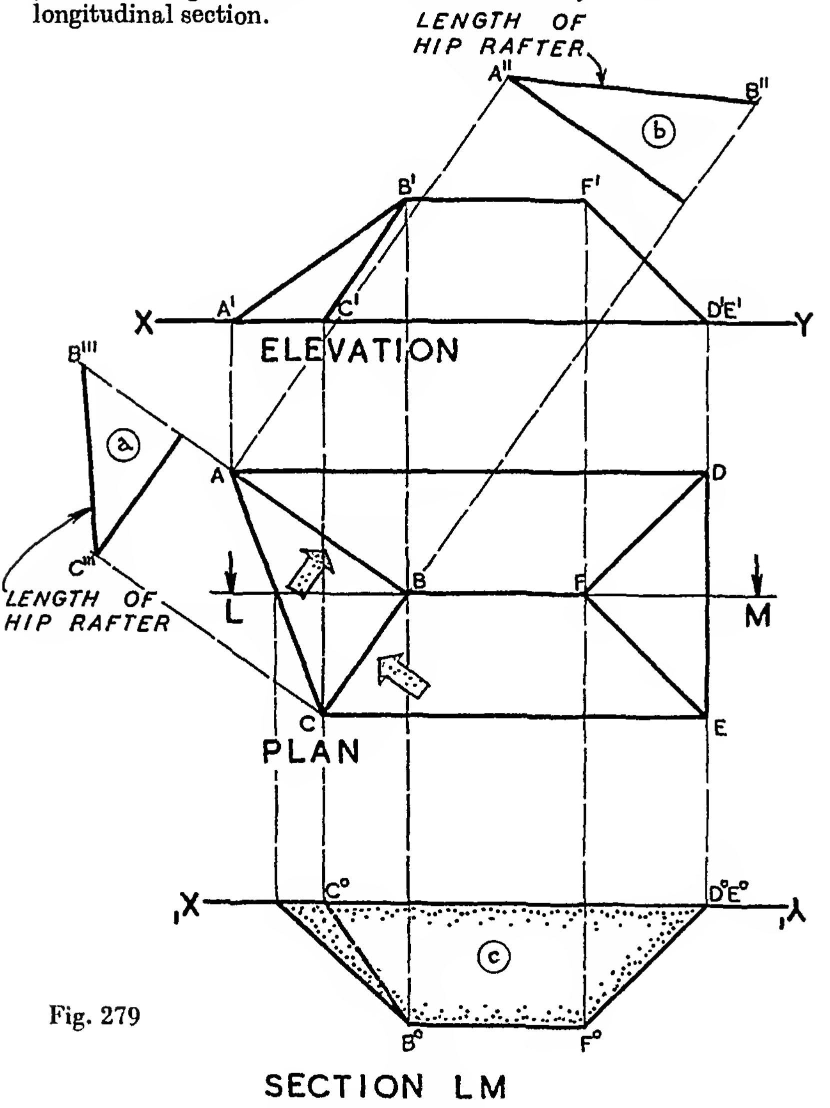


Fig. 279

Fig. 280 shows how the true plan of a skylight is obtained from the plan and sectional elevation.

Fig. 281 shows the elevations and plan of a carpenter's sawing stool and a new elevation at 60 degrees to the V.P. or when the new X–Y line is at 30 degrees to the original X–Y line.

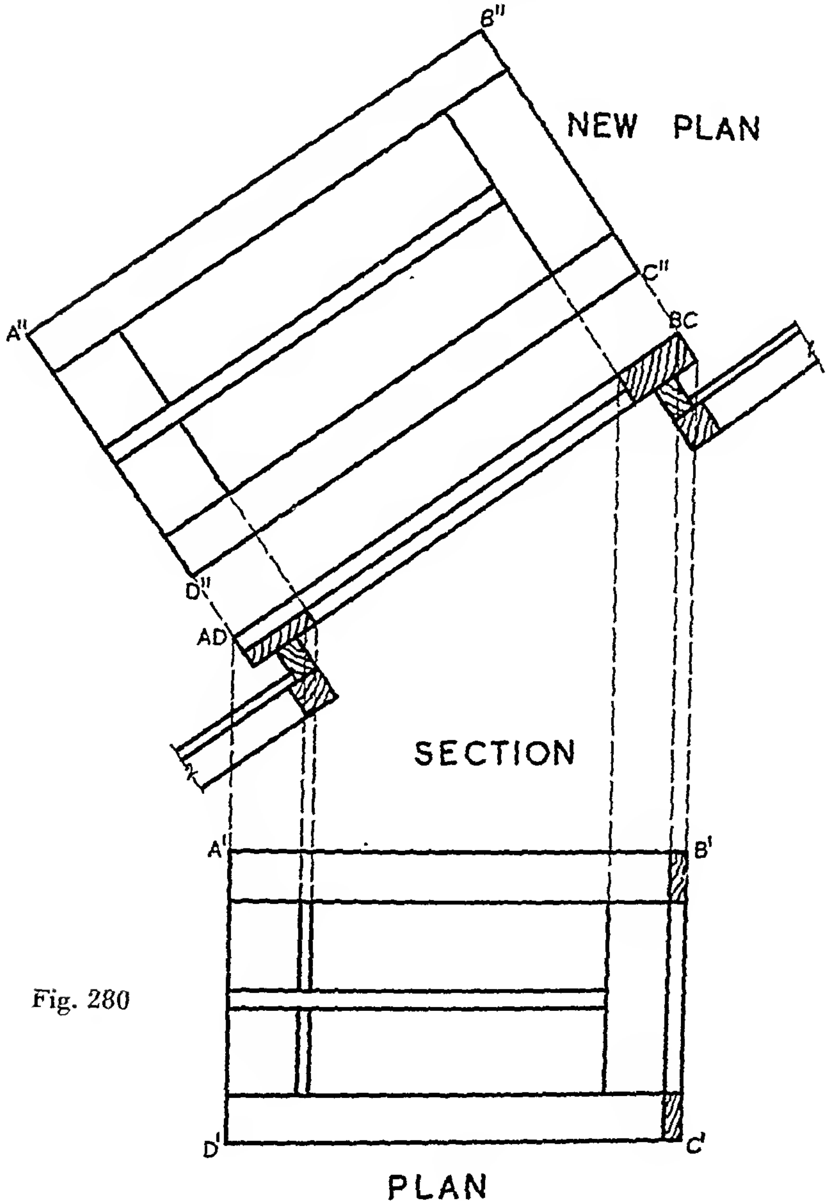


Fig. 280

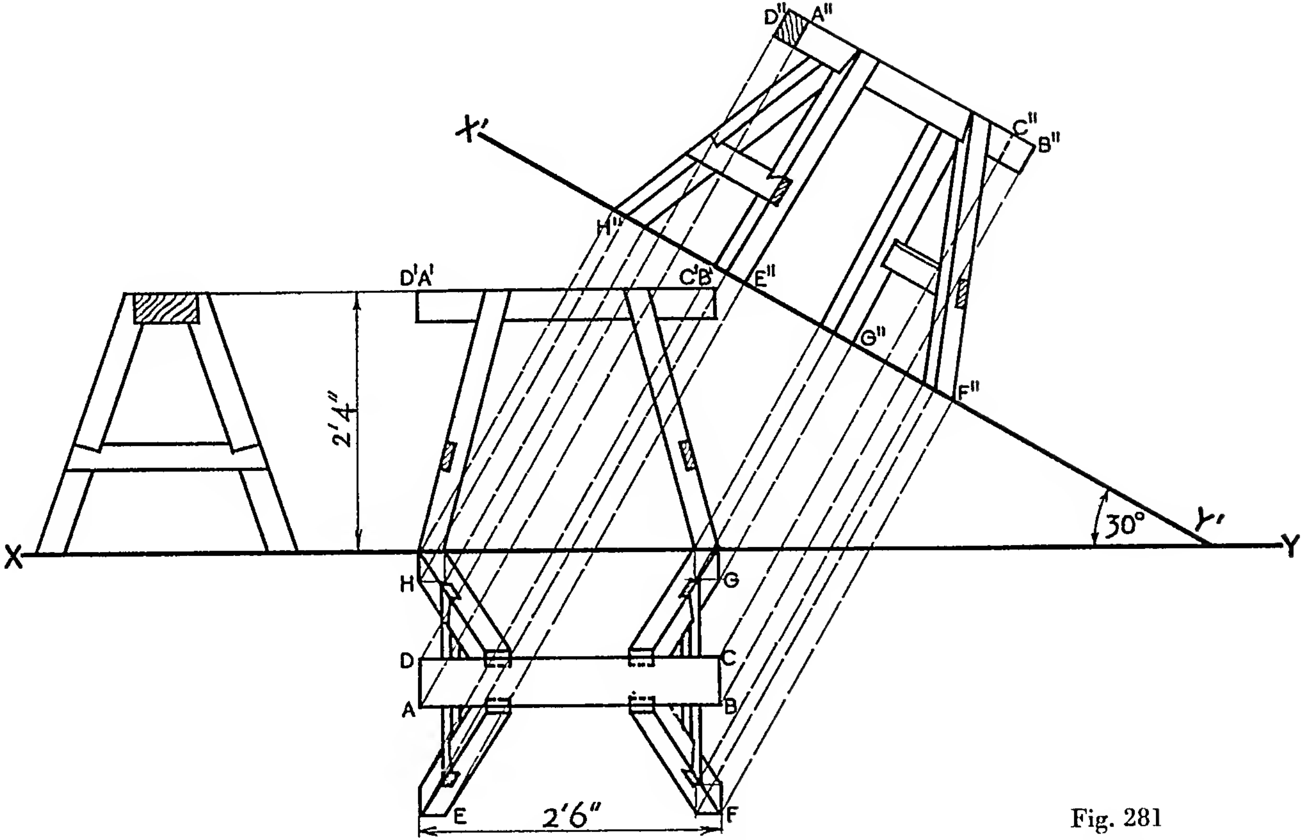


Fig. 281

Contouring

A contour is a horizontal level or cutting plane through the earth's surface and contour lines to show various heights above and below sea-level are shown on many maps and surveys. Fig. 282 illustrates a simple example showing the plan of a hill varying in height from 200 feet to 700 feet above sea-level. Contour lines are marked for every 100 feet. To obtain a section through the hill, a vertical cutting plane represented by lineXY is taken through the plan, and where intersections are made with contour lines, perpendicular ordinates are projected to contact corresponding co-ordinates parallel to the X–Y line; the sectional outline of the hill can then be traced. A new section on the line AB is also shown in the example.

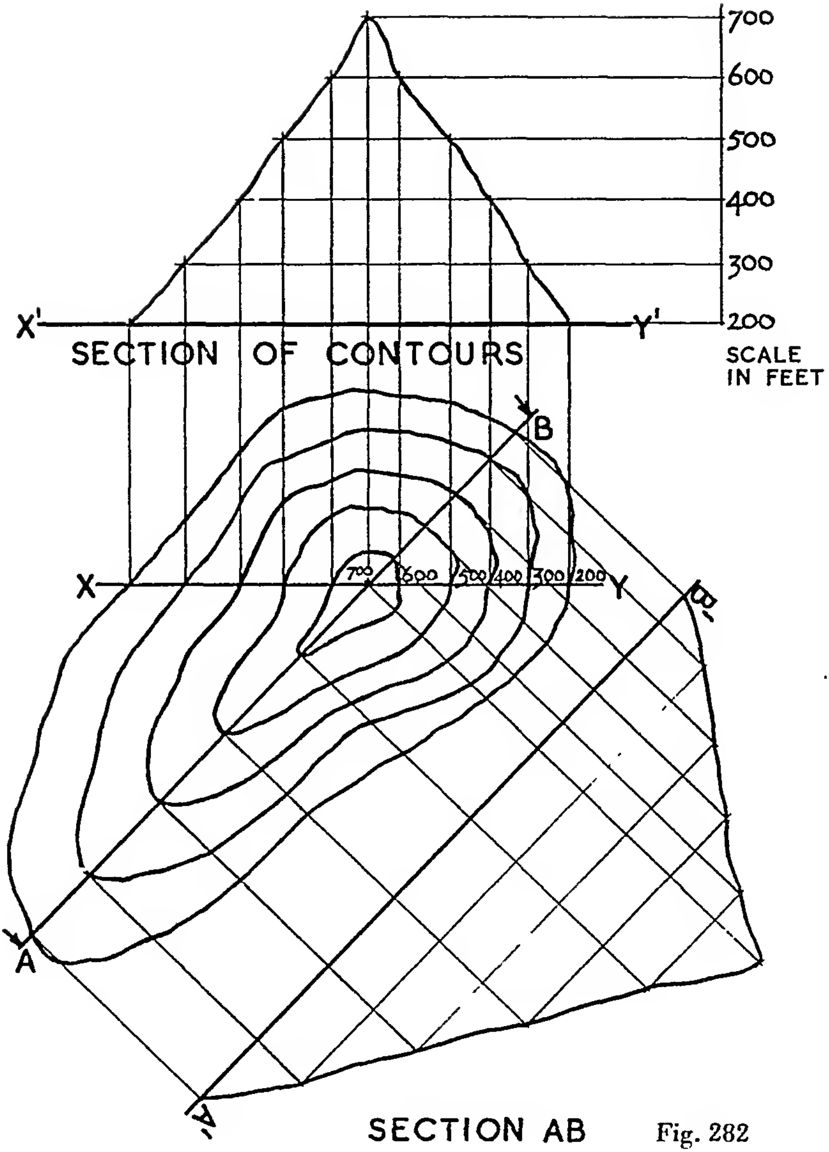

700
600
500
400
300
200
X'
Y'
SECTION OF CONTOURS
SCALE
IN FEET
B
X
Y
700
600
500
400
300
200
B'
A
A'
SECTION AB

Fig. 282

CHAPTER XIX

SECTIONS OF SOLIDS AND DEVELOPMENT OF SURFACES

PLOTTING SECTIONS OF SOLIDS CUT BY INCLINED PLANES

THE purpose of obtaining surface developments of solid objects is mainly to enable building operatives and estimating clerks to calculate the amount of material required to construct or cover various parts of a building. Although the subject can include complex and intricate forms, the following examples deal with comparatively simple basic developments. In addition to those given in the previous chapter, further examples of cutting planes are illustrated.

The Prism

Fig. 283 shows an octagonal prism cut by an inclined plane at an angle of 30 degrees to the H.P. The section of the cut and the development of the remaining portion of the prism is found as follows: The prism is drawn in plan and elevation and is indexed. From the V.T. perpendicular projectors are taken on which are plotted the distances on plan of the various points from the X–Y line. By joining the points so obtained the section of the cut is found.

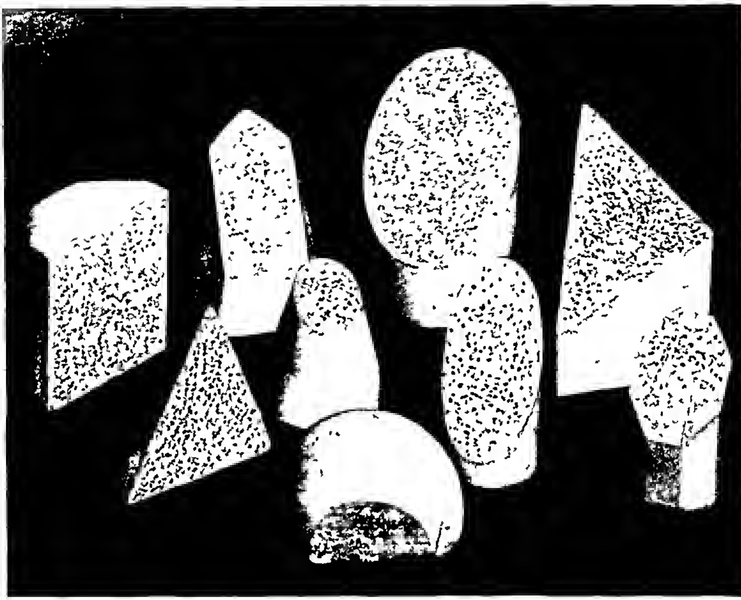
The development is obtained by plotting the perimeter in indexed lengths along the X–Y line extended. Perpendicular ordinates are then set up from A^1 to A'' to contact corresponding horizontal co-ordinates projected from the V.T.

The Cylinder

Fig. 284 shows a cylinder standing with its base on the H.P. and its axis parallel to the V.P. It is cut by an inclined plane at an angle of 45 degrees to the H.P.

The plan and elevation are drawn, and the circle on plan is divided into eight equal parts, indexed A to H. By projecting ordinates to the V.T. from these points and then proceeding as described for the foregoing, the section of the cut—an ellipse—is obtained.

In finding the development, the circumference of the circle should be plotted along the X–Y line extended. An approximation can be made by plotting the chords, AB, BC, and so on. The procedure is then as previously described.



CUT GEOMETRICAL SOLIDS

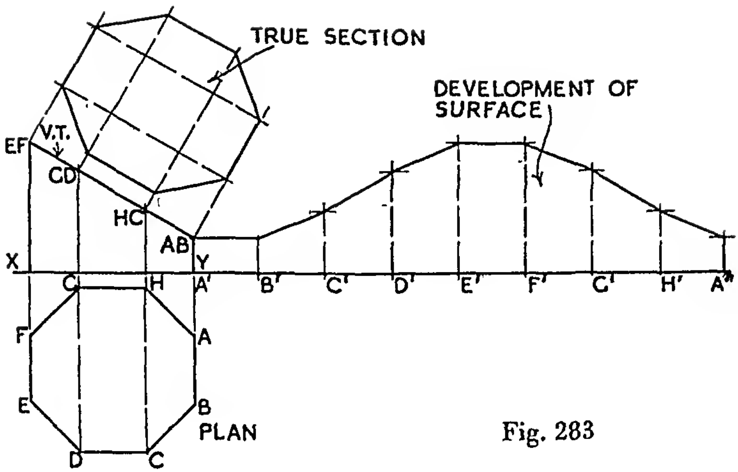


Fig. 283

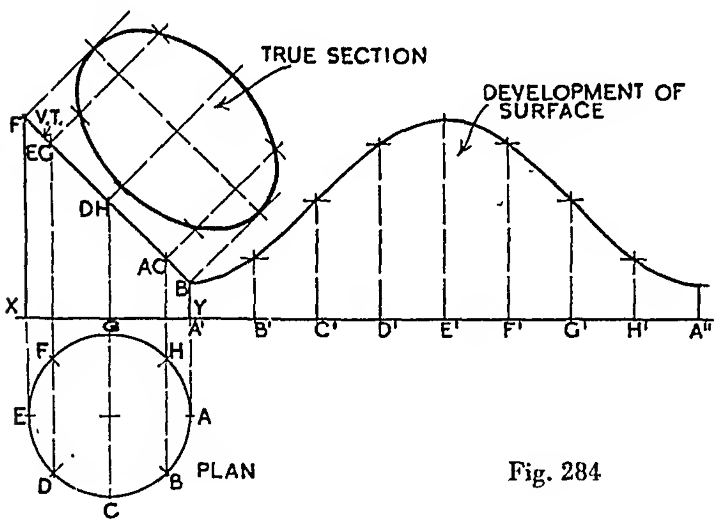


Fig. 284

The Pyramid

Fig. 285 shows an hexagonal pyramid standing with its base on the H.P. cut by an inclined plane at an angle of 30 degrees to the H.P.

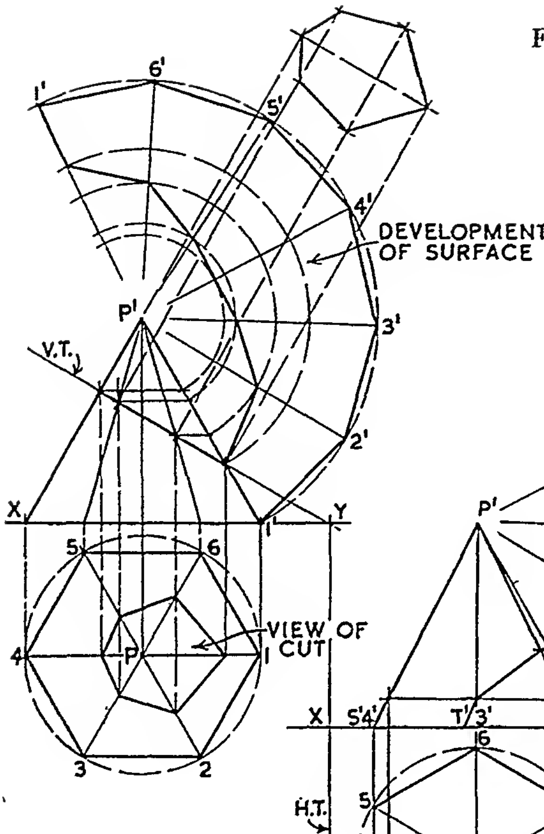
The plan and elevation are drawn and the position of the cut determined in elevation and projected down to the plan. The true section of the cut can then be obtained as before.

The development is found by taking centre P^1 and radius $P^1 1^1$ and describing an arc, along which is plotted the indexed perimeter at the base of the pyramid. Horizontal ordinates are taken from the cut in elevation to $P^1 1^1$ and continued as arcs with centre P^1 to contact co-ordinates drawn from 1^1, 2^1, 3^1, etc., to P^1.

Fig. 286 shows an hexagonal pyramid standing with its base on the H.P. and its base edges inclined at an angle of 30 degrees to the V.P. It is cut by a plane perpendicular to the H.P. and inclined at angle of 45 degrees to the V.P.

The plan and elevation are drawn and the position of the cut is determined on plan and projected to the elevation. The shape of the cut is found by the method employed for the previous examples. Note that the true length of $P^1 1^1$ must be found before the developed surfaces can be obtained.

Fig. 285



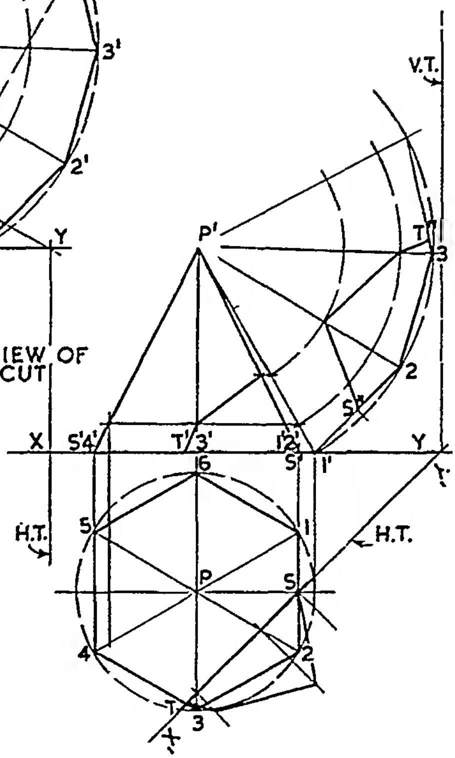


Fig. 286

The Hemisphere

Fig. 287 shows a hemisphere cut by a plane perpendicular to the H.P. and inclined at an angle of 45 degrees to the V.P. The plan and elevation of the hemisphere are drawn and the position of the cut is determined on plan. With centre *V* and radius tangent to the cutting plane in plan an arc is described to the horizontal axis, from which point is projected a perpendicular to the circumference of the hemisphere in elevation, this gives point 2. Another point, 1 (or more depending on the size of the drawing), is taken on the arc of the hemisphere and projected down to the horizontal axis on plan and continued as an arc, centre *V*, to contact the cutting plane. From the contact points on the cutting plane ordinates are projected back to the elevation to contact corresponding horizontal co-ordinates. The curve—semi-ellipse—of the cut in elevation can then be drawn through the points obtained. The true shape of the cut is found by producing a new elevation of the cut perpendicular to the H.T. as shown.

Fig. 288 shows a hemisphere cut by a plane perpendicular to the V.P. and at an angle of 45 degrees to the H.P. The view of the cut in plan and the true shape of the cut are found by methods similar to the foregoing.

It will be seen that cuts taken through a sphere or hemisphere are circular or semicircular in outline, and therefore the views of them when seen obliquely in plan or elevation are ellipses or semi-ellipses.

Fig. 289 shows a hemisphere cut by an oblique plane at angle of 45 degrees to the H.P. and to the V.P. The true shape of the cut is known to be a circle. The diagram illustrates the application of the previous methods in finding the views of the cut in plan and elevation. It is necessary to draw the true inclination of the cut in elevation for convenience of projections to the plan.

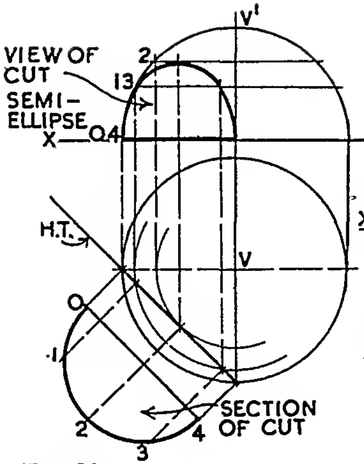


Fig. 287

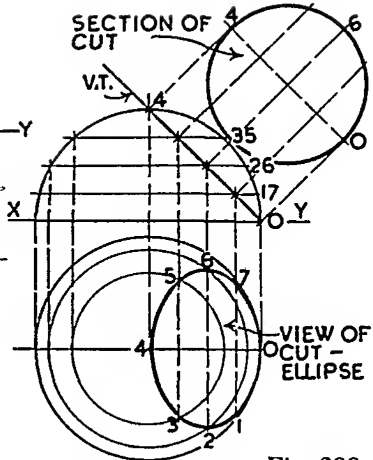


Fig. 288

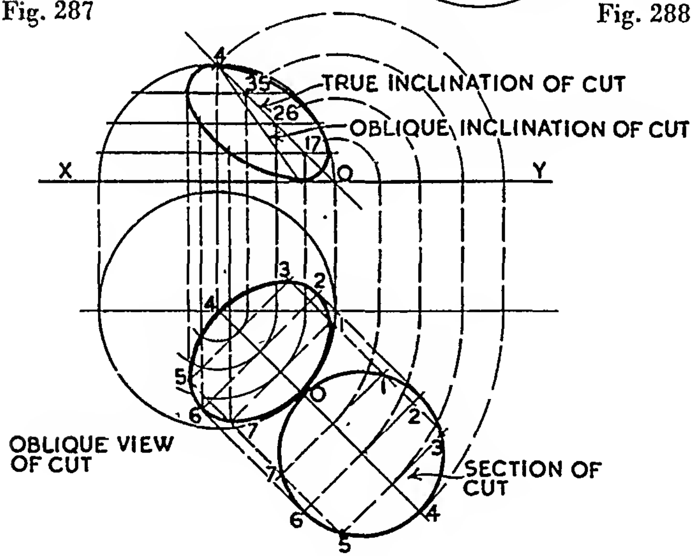


Fig. 289

The Cone

A cone may be cut by planes so as to give sections of different shape. Fig. 290 shows the traces of possible cutting planes:

A — horizontal cutting plane . . . circular section
B.— vertical cutting plane . . . hyperbolic section
C — vertical cutting plane through perpendicular axis . . . triangular section
D — inclined cutting plane, not parallel to side of cone . . . elliptical section
E — inclined cutting plane, parallel to side of cone . . . parabolic section

Fig. 291 shows a cone cut by a vertical cutting plane (as case B). Also shown is the trace of the cutting plane on the surface and the development of the surface.

The plan and elevation of the cone are drawn and the position of the cut on plan is determined. The circumference of the base on plan is marked with 12 equi-distant points, A to M, which are joined to the vertex V. A to M are projected to the base line in elevation and joined to the vertex V^1. The points of intersection on plan of the cutting plane and the lines VF, VE, VC and VB are projected to the elevation. With centre V and radius tangent to the cutting plane on plan an arc is described to the horizontal axis GA, and continued as a perpendicular to the side of the cone in elevation, from which point a horizontal line is drawn to cut the vertical axis at 4. Through the points now plotted on the elevation the curve of outline of the section can be drawn.

Additional arcs with centre V—two in this case—are drawn to intersect the cutting plane on plan. From the points of intersection lines are drawn radial from V to the circumference to give points 2 and 3, and perpendicular projections are made to the outline of the section in elevation to give points 2^1 and 3^1. From 2^1 and 3^1 horizontal lines are drawn to the side of the cone. With centre V^1 arcs are now described from the intersections along the side of the cone, and from A^1 the distances, AB, BC, CD, etc., are plotted and also the points 1, 2, 3 and 4. By joining A^1, B^1, C^1, D^1, etc., to V^1 the surface of the cone is developed, by drawing ordinates from 1, 2, 3 and 4 to cut corresponding co-ordinate arcs the trace of the cutting plane can be plotted.

Fig. 292 shows a cone cut by a plane parallel to the side of the cone (as case E, Fig. 290). Also shown is the section of the cut—the parabola—and the development of the surface showing the trace of the cutting plane.

The methods of finding the shape of the cut and the development are similar to those employed in the foregoing example and can be followed in the diagram.

Fig. 291

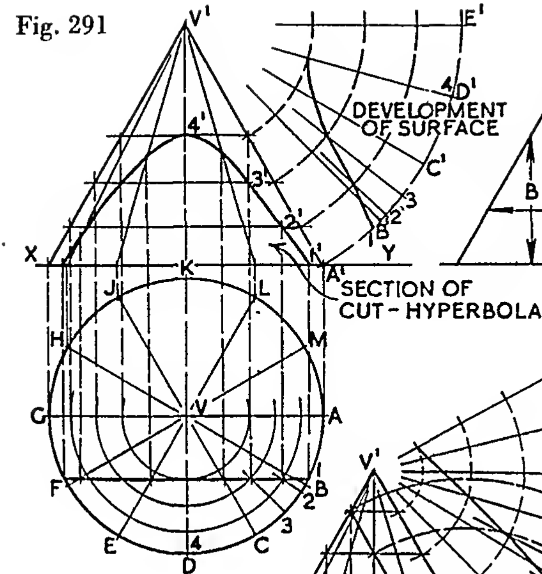
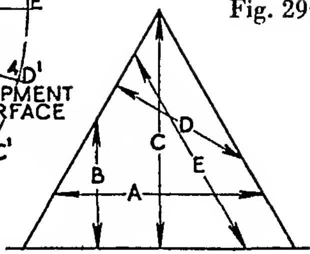


Fig. 290



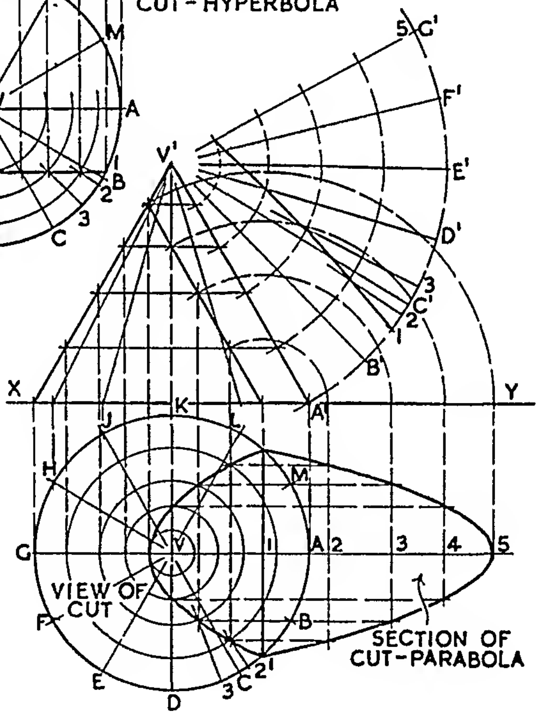


Fig. 292

The following illustrations (Figs. 293 to 300) are further examples of developed surfaces.

Fig. 293 shows a square pyramid having four sloping sides, each of which is an isosceles triangle. It will be seen that the length of the side *C′O′* in elevation is the radius of an arc, centre *O′*, on which the four equal base sides can be plotted to find the development.

Regular Polyhedrons

1—Tetrahedron	— figure with	four sides, each an equilateral triangle
2—Hexahedron	— ,, ,,	six sides, each a square (cube)
3—Octahedron	— ,, ,,	eight sides, each an equilateral triangle
4—Dodecahedron	— ,, ,,	twelve sides, each a regular pentagon
5—Icosahedron	— ,, ,,	twenty sides, each an equilateral triangle

Fig. 294 shows the plan, elevation and developed surface of the tetrahedron. Note: width of base, *BC*, governs the sides of the equilateral triangles.

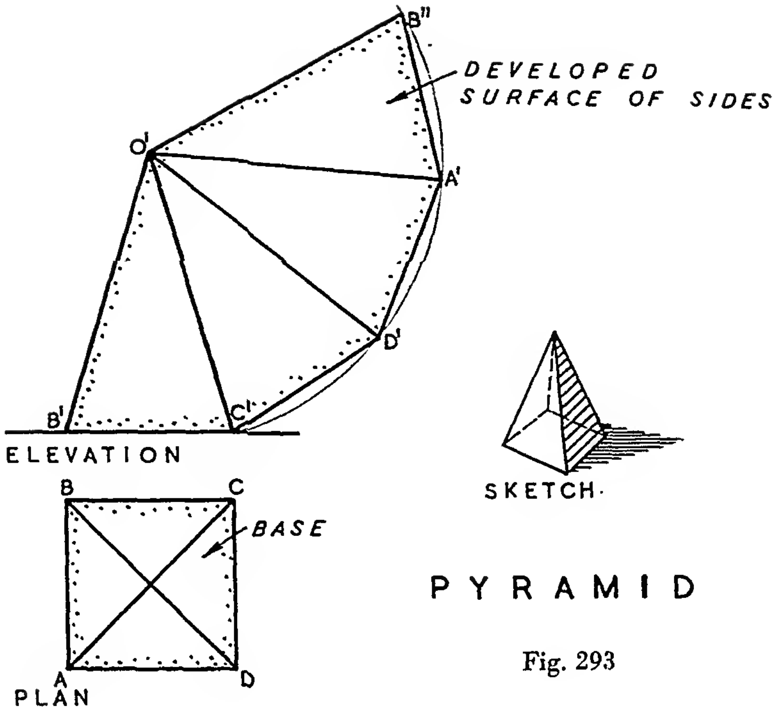
Fig. 295 shows the plan, elevation and developed surface of the hexahedron. The surface development may be found in other ways.

Fig. 296 shows the development of the surface of the dodecahedron.

Fig. 297 shows the plan, elevation and developed surface of the octahedron. Note: width of base, *BC*, governs the sides of the equilateral triangles.

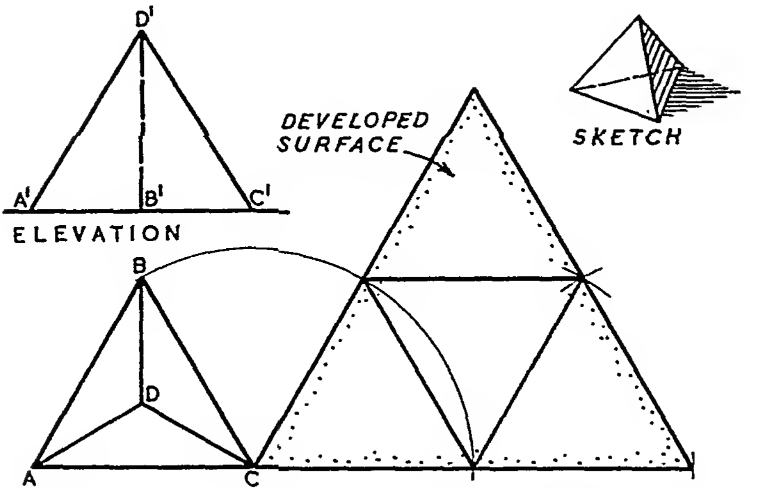
Fig. 298 shows another method of developing the surface of the octahedron.

Fig. 299 shows the development of the surface of the icosahedron.



PYRAMID

Fig. 293



TETRAHEDRON

Fig. 294

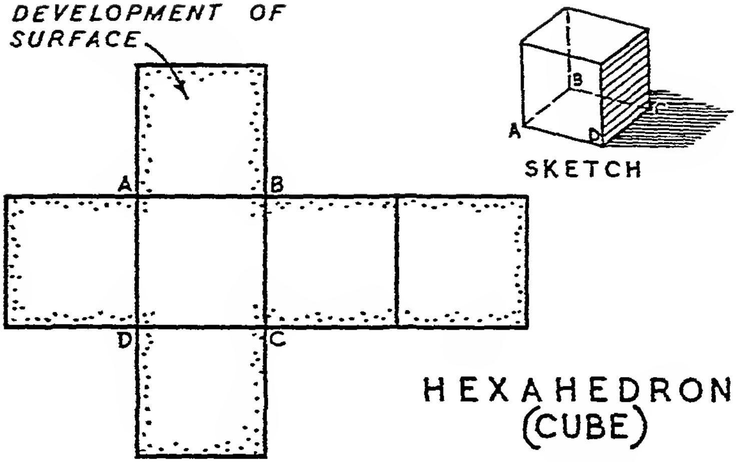
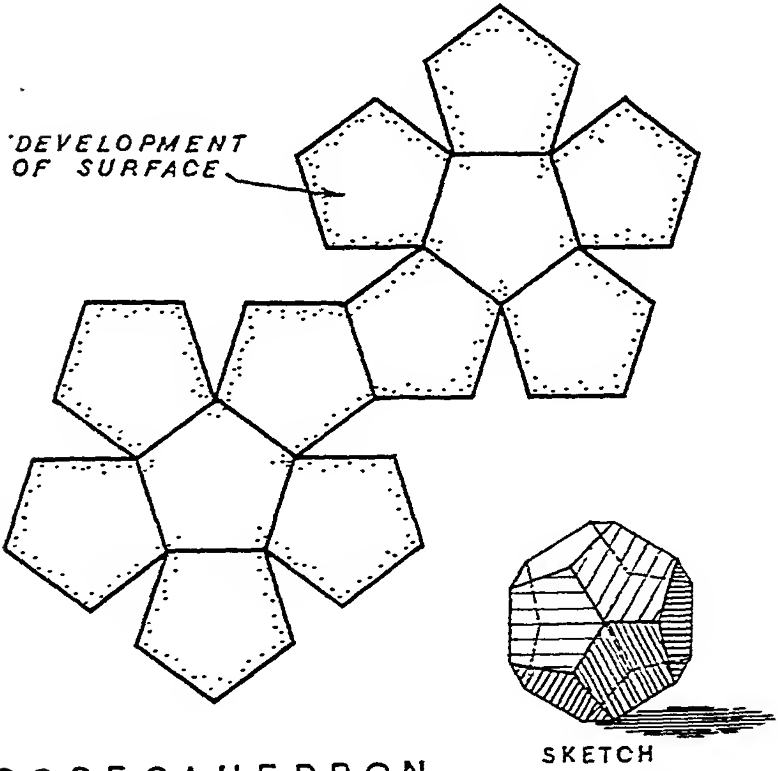


Fig. 295



DODECAHEDRON

Fig. 296

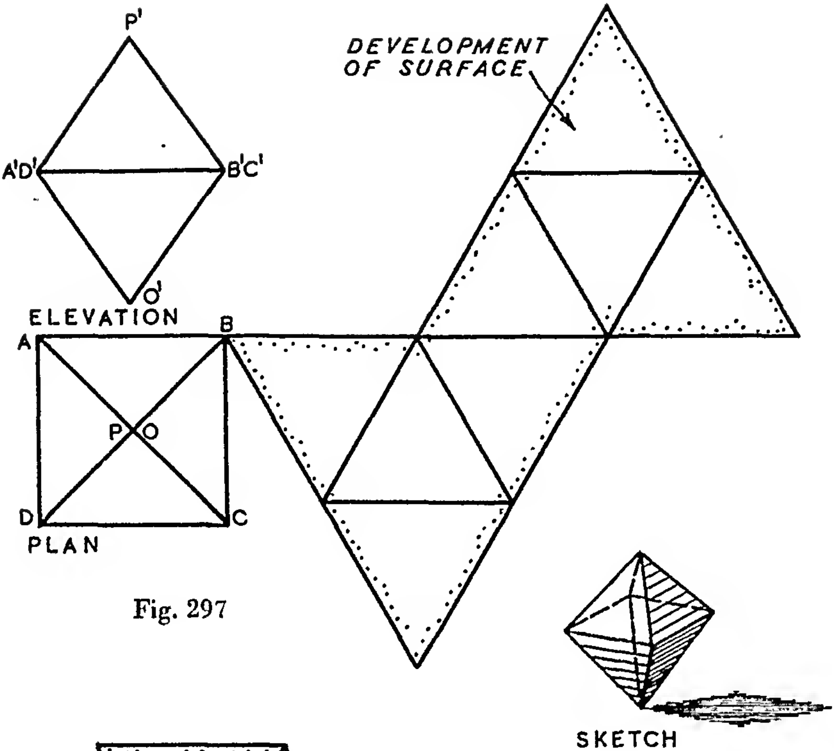


Fig. 297

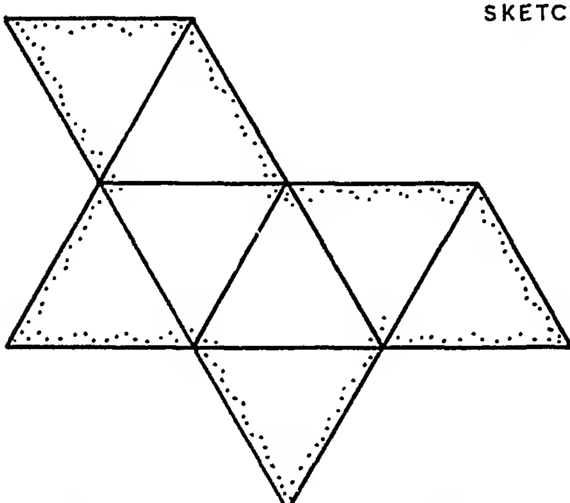
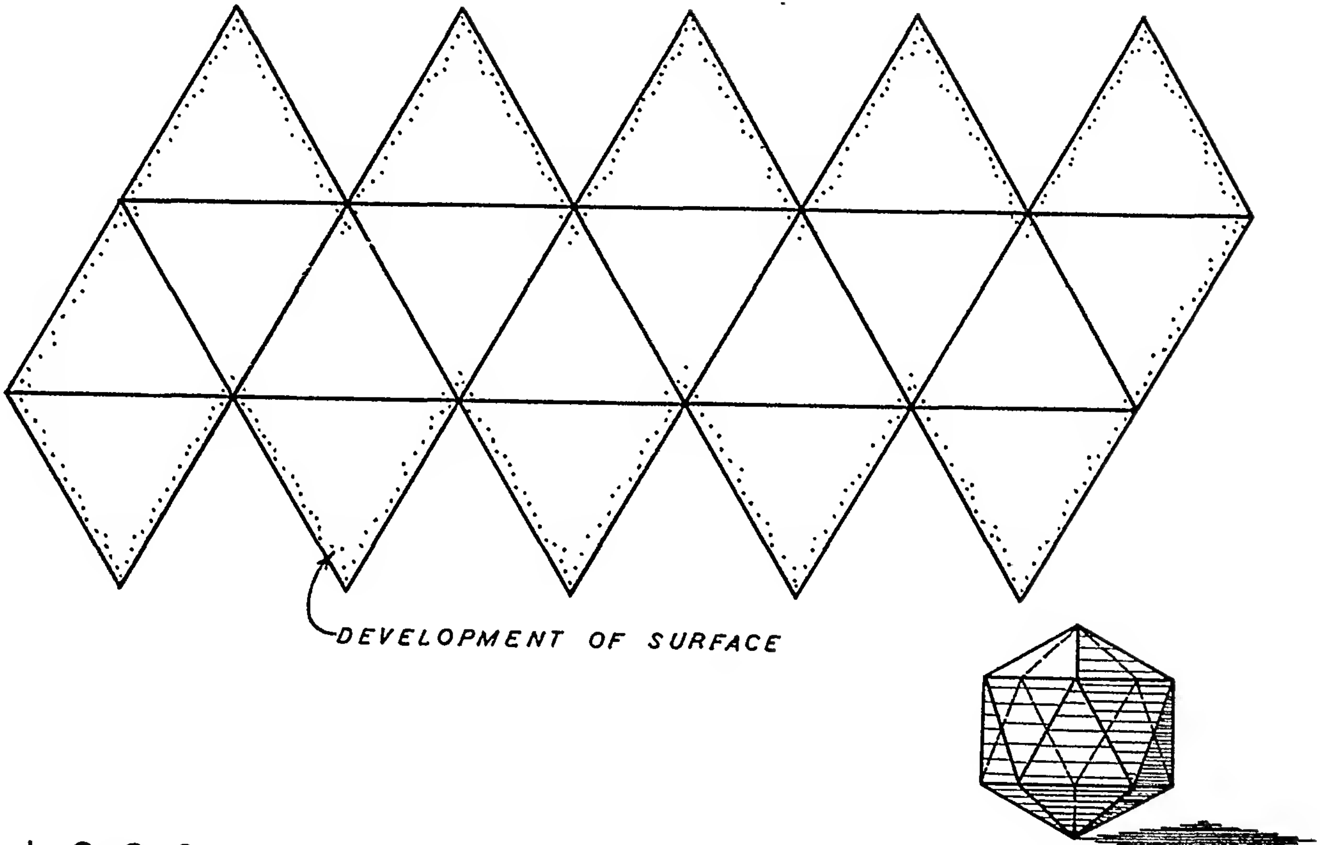


Fig. 298

OCTAHEDRON



ICOSAHEDRON

Fig. 299

Splayed Lining

Fig. 300 shows a straightforward example of a splayed head-lining to a semicircular opening. The lining is shown in plan and elevation, with a development of its surface. The surface is part of a cone, the apex of which is R′ on plan. To find the development: with centre R' and radii $R'P'$ and $R'O'$ describe arcs as shown. On the outer arc set out from O' units obtained by dividing the corresponding semicircle on elevation into a convenient number of equal parts. By joining the divisions radially to the inner arc the developed surface or any part of it is obtained. (Compare Figs. 290 and 292.)

Conoid

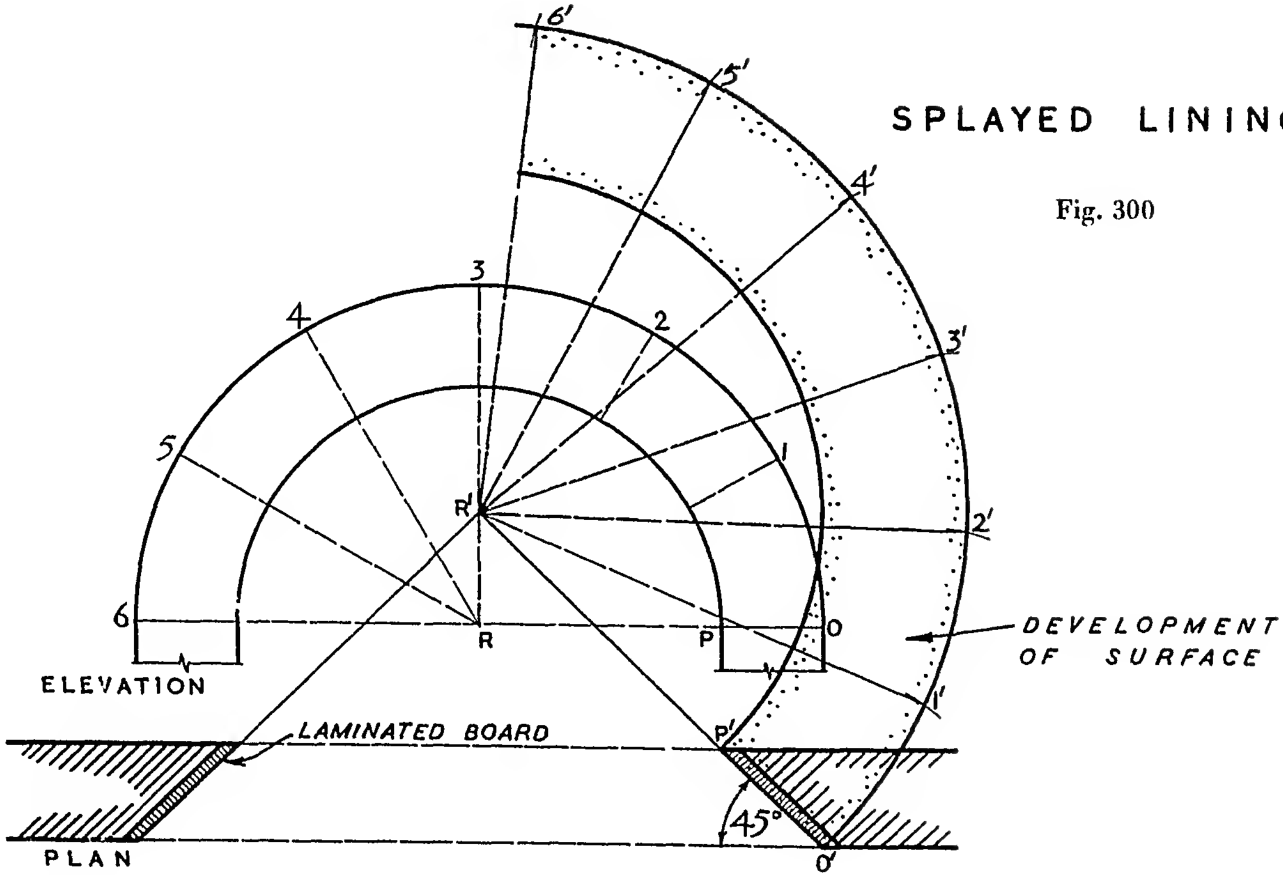
The shape of the conoid is illustrated by the end of a stream-lined motor body, or the bows of a boat, but in building it occurs mainly in some semicircular door and window openings. Other practical applications are to be found in advanced construction and geometry books, but at this stage it is sufficient to know how the development is obtained. (Note: it is not possible to obtain an accurate development of the surface, but the method given below is the nearest approximation.)

Fig. 301 shows the conoid in plan and elevation, and diagrammatic sketch. The plan is a triangle, and the elevation is represented by a semicircle or semi-ellipse, according to where the axis is cut. From the sketch it will be seen that the height of the figure is the same at the "front" and the "back", i.e. $P6$ equals $R6'$.

To develop the surface, divide the "front" of the conoid in elevation into a number of equal parts—12 in this case—project to the plan and radiate lines to R (all these lines are true lengths). At R set off RA equal to $O'1'$, at right angles to RO'' With centre A and radius equal to $R1$ describe an arc to cut at $1''$ an arc drawn with centre O'' and radius equal to $O1$ in elevation. Join A and O'' to $1''$. Obtain line $B2''$ in a similar manner, AB being equal to $1'2'$, and $1''2''$ equal to 12 in elevation, and continue to complete the development.

SPLAYED LINING

Fig. 300



Pipe Intersections and Developments

Where lead and other pipes intersect at various angles, careful

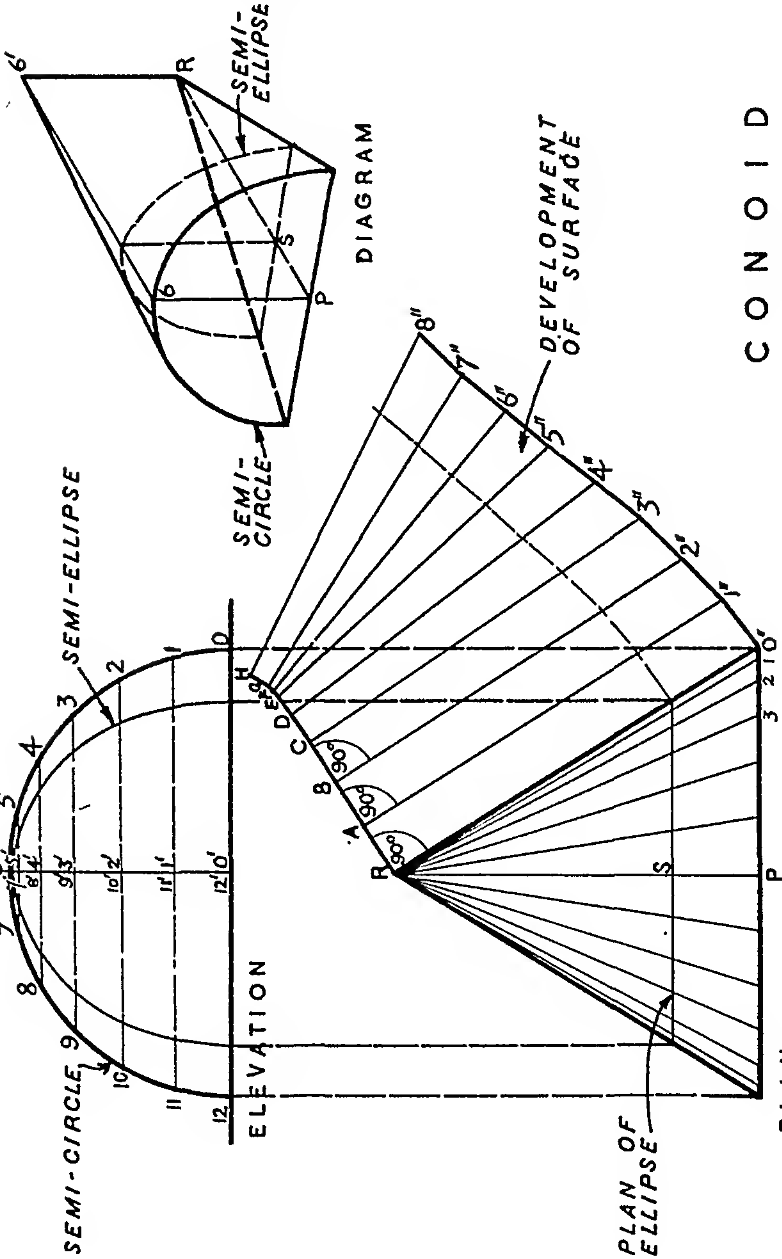


Fig. 301

jointing is necessary, and it is important that the material is cut very accurately. The following three examples give the general procedure for practical purposes.

Fig. 302 shows two intersecting cylindrical pipes. The axes of both pipes are in the same plane. The line of intersection is obtained by marking a number of points about the section of the pipe seen in elevation, projecting ordinates from them to the plan to intersect co-ordinates projected from the superimposed section of the interpenetrating pipe.

The developed surface of the interpenetrating pipe is found by plotting its circumference with indexed points on a line at right-angles to its axis, and from these points producing ordinates parallel to the axis to contact co-ordinates drawn from the joint in plan as shown.

Fig. 303 shows a pipe intersected by a smaller pipe with the axes in the same horizontal plane but inclined to one another at an angle of 45 degrees on plan. The line of intersection is found by the method used for the previous example, as is the developed surface of the smaller pipe. It will be observed that the developed surface of any parallel rectilinear figure unrolls, as it were, at right-angles to the axis of the figure.

Fig. 304 shows a pipe intersected by a smaller pipe with the top surfaces of both pipes level and in the same plane, but the axes on plan are inclined to one another at an angle of 45 degrees. The line of intersection and the developed surface of the smaller pipe are found by the methods used for the previous examples.

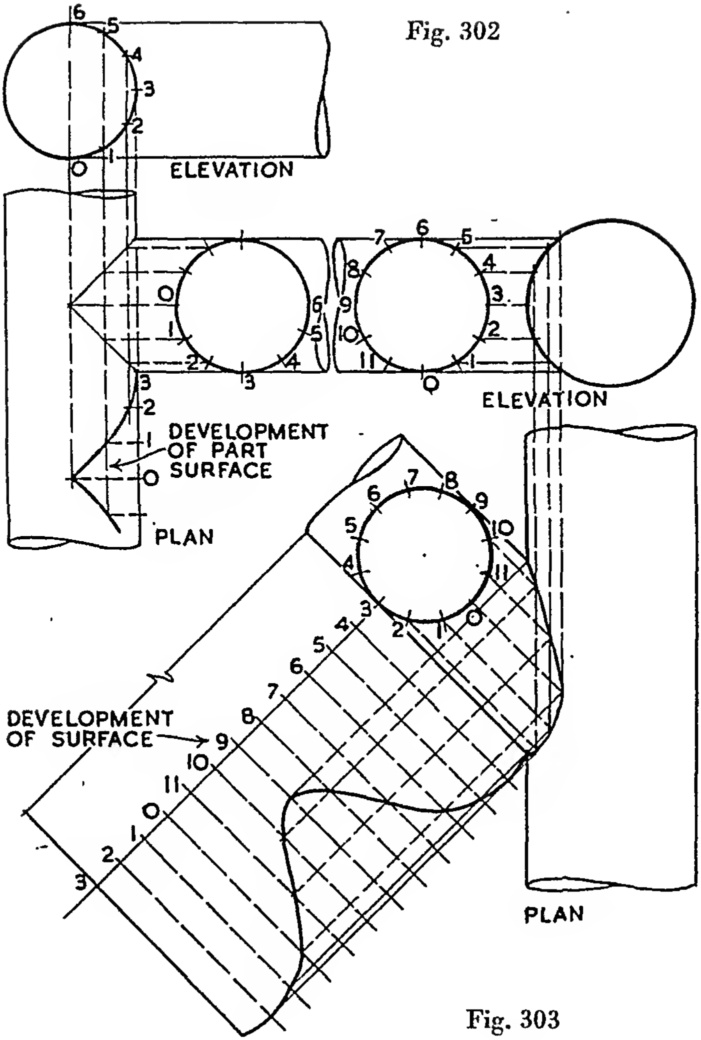


Fig. 302

Fig. 303

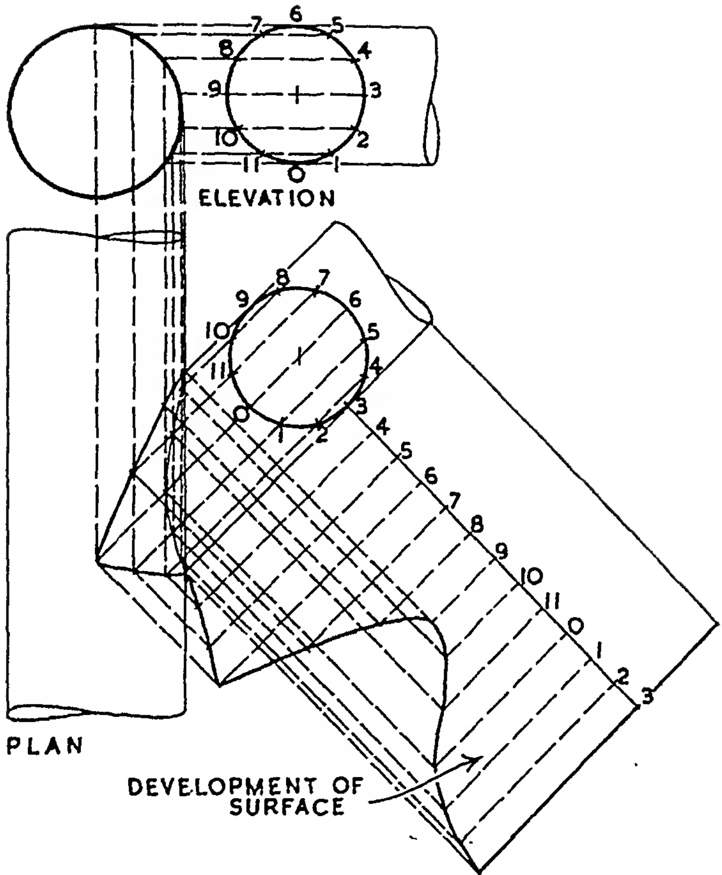


Fig. 304

QUESTIONS

EXERCISE 1

Scales (see Chapter III)

1. Draw a linear scale of $1\frac{1}{2}$ inches=1 foot showing inches, $\frac{1}{2}$ and $\frac{1}{4}$ inches and to measure a length of 4 feet.

2. Draw a diagonal scale of 1 inch=10 feet to show feet and inches and to measure a distance of 50 feet. Indicate a length of 47 feet 11 inches on your scale.

3. Draw a diagonal scale of 1 inch=1 yard to show feet and inches and to measure a length of 5 yards. Mark on your scale a length of 4 yards 2 feet 7 inches.

4. Draw a diagonal scale of $1\frac{1}{2}$ inches to the mile showing furlongs and chains and to measure 5 miles. Show on your scale a distance of 3 miles 5 furlongs 7 chains.

5. Draw a diagonal scale of 1 inch to the chain showing feet and to measure a distance of 5 chains.

6. Draw a diagonal scale in which the representative fraction is $\frac{2}{5}$, showing centimetres and millimetres and to measure a total length of 30 centimetres.

7. A line $2\frac{1}{4}$ inches long is known to represent a length of 15 inches. Make a scale to read feet and inches and to measure a length of 3 feet.

EXERCISE 2

Lines, Angles and Triangles (see Chapter IV)

1. The perimeter of a triangle equals 7 inches and the sides are in proportion of 3 : 4 : 5. Obtain the lengths of the sides geometrically and construct the triangle.

2. (*a*) On a base line $1\frac{1}{2}$ inches long construct an isoceles triangle in which the angle at the apex is 30°.

(*b*) On the same base line construct a right-angled triangle (using your compasses and straight edge only) equal in area to the isoceles triangle drawn in (*a*).

3. (*a*) Draw an equilateral triangle in which the inscribed circle is $2\frac{3}{4}$ inches diameter.

(*b*) Draw an equilateral triangle in which the circumscribing circle is $3\frac{1}{2}$ inches diameter.

4. (*a*) Construct a right-angled triangle in which the longest side is $3\frac{1}{2}$ inches and the shortest side is $1\frac{1}{2}$ inches long.

(*b*) Construct a right-angled triangle in which the longest side, $3\frac{1}{2}$ inches long, is the base and the vertical height is $1\frac{1}{2}$ inches.

(*c*) Construct a right-angled triangle, which is also isoceles, and the longest side $3\frac{1}{2}$ inches long.

5. Draw right-angled triangles in which the sides are in the proportion of (*a*) $3:4:5$. (*b*) $5:12:13$ and (*c*) $8:15:17$, the shortest side in each case being 2 inches.

6. Draw the three triangles similar to those in Question No. 5 but in each case having an inscribed circle of $1\frac{1}{4}$ inch radius.

7. Draw the three triangles similar to those in Question No. 5 but in each case having a circumscribed circle of $3\frac{1}{2}$ inches diameter.

EXERCISE 3

Quadrilaterals and Parallelograms (see Chapter V)

1. Draw a square in which the length of the diagonals is 3 inches. Measure and write down the length of the sides.

2. Draw a rhombus in which the lengths of the diagonals are 3 inches and 2 inches respectively. Measure and write down the lengths of the sides and the angles between adjacent sides.

3. Draw a rectangle in which the ratio of the sides is $3:4$ and the diagonals $3\frac{1}{2}$ inches long.

4. The diagonals of a parallelogram are $3\frac{1}{2}$ inches and $2\frac{1}{2}$ inches long and the shortest side is $2\frac{1}{4}$ inches long. Draw the parallelogram and measure and write down the smaller of the angles between the diagonals.

5. Draw the quadrilateral $EFGH$ in which $EF=2\frac{3}{4}$ inches, $FG=2\frac{1}{2}$ inches, $EG=3\frac{1}{4}$ inches, $GH=2\frac{1}{2}$ inches and angle $GHE=90°$.

6. Draw the quadrilateral $ABCD$ in which $AB=2\frac{1}{2}$ inches, $BC=$ 2 inches, angle $ABC=105°$, $CD=2\frac{1}{4}$ inches and $AD=2\frac{1}{2}$ inches.

7. A field in the shape of a quadrilateral has the following lengths of sides. $AB=550$ yards, $BC=465$ yards, $CD=385$ yards, $CA=430$ yards. The diagonal $BD=635$ yards. Draw the shape of the field to a scale of 1 inch$=200$ yards and measure and write down the length of the diagonal AC.

EXERCISE 4

Polygons (see Chapter VI)

1. Using your tee-square and set-square only, draw on a line $1\frac{1}{2}$ inches long.

(a) A regular hexagon.
(b) A regular octagon.

2. (a) Draw a regular hexagon in which the circumscribing circle is $3\frac{1}{2}$ inches diameter.

(b) Draw a regular octagon in which the diagonals are 4 inches long.

3. (a) Draw a regular hexagon in which the distance between parallel sides is 3 inches.

(b) Draw a regular octagon in which the inscribed circle is 3 inches.

4. Draw a regular pentagon of 2 inches side; measure and write down the angle between two adjacent sides.

5. Using the pentagon drawn in Question 4 (a) enlarge it so that the diagonals are 4 inches long and, (b) reduce it so that the diagonals are 2 inches long.

6. Repeat Questions 4 and 5 but substituting the word heptagon for pentagon.

7. Draw (a) A regular pentagon and (b) a regular heptagon, in each case of which the circumscribing circle is 4 inches diameter. (*Hint:* these must be done by drawing a regular pentagon and regular heptagon of any dimensions and enlarging or reducing these figures to satisfy the dimensions of 4 inches.)

EXERCISE 5

The Circle (see Chapter VII)

1. Draw (a) a circle of 1 inch radius and (b) a circle of twice the area that drawn in (a). (*Note:* the areas of similar figures vary as the square of their similar dimensions.)

(*Hint:* the radius of the second circle will be

$$\frac{\text{one} \times \text{hypotenuse of } 45^\circ \text{ set square}}{\text{length of side of } 45^\circ \text{ set square}}\text{)}$$

2. Draw a circle of $1\frac{1}{2}$ inches radius and on the same centre another circle of (a) twice the area, (b) four times its area, and (c) one-half its area.

3. Draw a circle of $1\frac{1}{2}$ inches radius, (a) a concentric circle of 3 times its area and (b) a circle of $\frac{1}{3}$ its area.

4. Draw a circle of $1\frac{1}{2}$ inches radius and divide its circumference into 16 equal divisions; join these points to the centre of the circle thus obtaining 16 approximate triangles. Obtain the approximate area of the circle.

5. Draw a circle, centre O, of radius $1\frac{1}{4}$ inches and mark off $OB=$ 3 inches horizontally, letter the point of intersection of OB and the circumference of the circle A. Obtain points C and D, point C being 2 inches from both A and B, and point D $2\frac{1}{4}$ inches from A and $2\frac{3}{4}$ inches from B

6. Draw two circles of centres A and B of radius $1\frac{1}{4}$ inches and $1\frac{3}{4}$ inches respectively. $AB=2\frac{1}{4}$ inches, now obtain a point C which is 2 inches from the circumference of each of the circles drawn. Draw the circle, centre C, of 2 inches radius.

7. Draw a circle of $2\frac{1}{2}$ inches radius touching the circles of centres A and B in Question 6.

8. Draw a circle, centre O, of $1\frac{1}{2}$ inches radius. Obtain the positions of points A and B so that $OA=3\frac{1}{4}$ inches, $OB=2\frac{3}{4}$ inches, and $AB=$ $1\frac{3}{4}$ inches. Now obtain a point C which is equidistant from A and B and the circumference of the circle.

9. Repeat Question 8 but making $OA=OB=AB=3$ inches.

EXERCISE 6

The Ellipse (see Chapter VIII)

1. The major axis of an ellipse is 4 inches long and its minor axis is 3 inches long. Draw the ellipse by three different methods: (*a*) the trammel method, (*b*) by concentric circles and radial lines and (*c*) by drawing a rectangle of sides equal to the major axis and lines from the extremities of the minor axis.

2. Trace the ellipse obtained in (*b*) of Question 1 and compare those obtained in (*a*) and (*c*) by placing the tracing over these ellipses.

3. Draw an ellipse of major and minor axes of $4\frac{1}{4}$ inches and $3\frac{1}{4}$ inches respectively and obtain the foci.

4. The major axis of an ellipse is $4\frac{1}{2}$ inches long and the distance between its foci is $3\frac{1}{4}$ inches. Draw the ellipse.

5. Draw one symmetric half of an ellipse in a rectangle 4 inches by $1\frac{1}{2}$ inches.

6. If you were required to set out a complete ellipse on a piece of land, the trammel method no doubt would be best, Give a clear lined

sketch to show the framing that would be needed, index the constructive points.

7. An elliptical stone arch with major axis as the span measures 6′ 0″ and the half minor axis as the rise measures 1′ 9″. Draw the outline of this arch to a scale 1 inch to 1 foot and show how you determine the correct joints of stones.

EXERCISE 7

Areas (see Chapter IX)

1. Draw on a line $2\frac{1}{4}$ inches long, an equilateral triangle and a right-angled triangle of the same altitude and the same area.

2. Draw an isoceles triangle the longest side of which is 3 inches long and the equal angles 45°. On the same base draw a rectangle equal in area to the isoceles triangle.

3. Draw a parallelogram whose sides are 3 inches and 2 inches long and the larger angles 105°. On the same base draw a rectangle equal in area to the parallelogram.

4. Draw a triangle whose sides are 3 inches, $2\frac{1}{2}$ inches and $2\frac{1}{8}$ inches long and on the same base an isoceles triangle of the same area.

5. Reduce the quadrilateral of Question 5 (Exercise 3) to :
   (*a*) a triangle of equal area,
   (*b*) a rectangle equal in area to the quadrilateral,
   (*c*) a square of equal area.

6. Reduce the quadrilateral of Question 6 (Exercise 3) to (*a*) a triangle, (*b*) a rectangle, (*c*) a square, all of the same area.

7. The representative fraction of the scale used in Question 7 (Exercise 3) is $\frac{1}{7200}$. Reduce the shape drawn of the field to a rectangle of equal area, and one inch high and calculate the area of the field (*a*) in square yards (*b*) in acres and square yards.

EXERCISE 8

Special Curves (Chapter X)

1. Draw a helix of $2\frac{1}{2}$ inches diameter having (*a*) a pitch of 1 inch and (*b*) a pitch of 2 inches.

2. Draw the locus of a point which moves so that its distance from a line AB is always equal to its distance from a point C. Point C being one inch from AB.

3. Obtain the position of a point E which is equidistant from the line AB and point C of Question 2 and is also the same distance from a point D. Point D is $2\frac{1}{2}$ inches from the line AB and $1\frac{1}{2}$ inches from C.

4. Two circles of radii $1\frac{1}{2}$ inches and 1 inch have their centres 2 inches apart. Draw the locus of the centres of all circles up to $1\frac{3}{4}$ inches radius which touch the two given circles.

5. A straight line of indefinite length is 2 inches from the centre of a circle of $1\frac{1}{4}$ inches radius. Draw the locus of the centres of all circles up to 2 inches radius which touch both the given line and given circle.

6. Draw parabolas in a rectangle 4 inches by 2 inches (*a*) where the greatest width is 4 inches and (*b*) where the greatest width is 2 inches.

7. A hyperbolic arch is to be constructed of brickwork its span being 5 feet and its rise being 2 feet 9 inches. Draw to the scale $\frac{3}{4}$ of one inch to 1 foot the outline of this arch. *Note:* joints of bricks need not be included.

EXERCISE 9

Outlines of Arches (see Chapter XI)

1. Draw to the scale 1 inch to 3 feet the outline of a semi-elliptical arch of 9 feet span and 3 feet 6 inches rise to be constructed in gauged brickwork. *Note,* leave all construction lines to show your method adopted in setting out.

2. Draw to the scale 1 inch to 4 feet the outline of a parabolic arch of 15 feet span and the rise being 6 feet.

3. Draw to the scale 1 inch to 2 feet the outline of a segmental arch to be constructed with stone, and to include a few joints. The span is 7 feet and the rise being 1 foot 9 inches.

4. Draw to any reasonable proportion the single outline of (*a*) 3 centred Gothic arch, (*b*) four centred Gothic arch.

5. Draw to a convenient proportion the single outline of a Venetian arch in Masonry. Give directions of a few stone joints which form this arch.

6. Draw to any convenient proportion the single outlines of any two arches you may know but of which are not included in the previous questions. In each case name the arch.

EXERCISE 10

Pattern (see Chapter XII)

1. Draw an equilateral triangle of 1¼ inches sides and on each side and outside the triangle describe a semicircle. Draw lines tangent to the semicircles and parallel to the given equilateral triangle.

2. Draw a square of 1½ inch side and on each side of the square and outside it describe a semicircle. Now draw a square so that each side is tangent to two of the semicircles.

3. Repeat Question 2, but each side of the second square is to touch only one (but different) of the semicircles.

4. Draw an equilateral triangle of 2 inch side and on each side and outside the triangle draw a semicircle of 1 inch radius. Now draw the circle touching, but not cutting each of the semicircles.

5. (*a*) Draw a regular pentagon of 1½ inch side and on each side and outside the pentagon describe semicircles of ¾ inch radius.

(*b*) Obtain the geometric centre of each pentagon and draw the circle tangent to each of the semicircles.

(*c*) Repeat (*a*) and draw the regular pentagon each side of which is tangent to a semicircle.

6. Repeat Question 5, but substituting a regular hexagon for the regular pentagon.

7. Draw a rectangle 2 inches by 1 inch, obtain the diagonal. Design any suitable pattern making use of the compasses and the diagonal as the diameter of a circle or part circle.

EXERCISE 11

Mouldings (see Chapter XIII)

1. Set out to any convenient scale the following detail Roman sections (*a*) Torus, (*b*) Scotia, (*c*) Cavetto, (*d*) Astragal.

2. An architrave moulding to be worked on 3½ inch by 1¼ inch timber is to embody the following detail, Cavetto, Ovolo, Cyma Recta or Cyma Reversa. Set out this section full size.

3. Two ovolo moulded architrave mouldings each measuring 3 inches by 1 inch intersect on the same plane, but form an angle of intersection at 120°. Draw to the scale full size and obtain the section of the mouldings at the mitre.

4. To any convenient proportion and scale design a simple cornice moulding for the eaves of a roof. The combination of mouldings may be "Roman" or "Grecian."

5. Referring to the previous question it may assume that this cornice is to be enlarged in proportion to be $\frac{1}{3}$ larger. Show geometrically what method you would adopt to arrive at the result.

6. The architrave moulding referred to in Question 2 has to be reduced from $3\frac{1}{2}$ inches in width to $2\frac{3}{4}$ inches, but its thickness is to be maintained. Show to scale=full size, what geometric method you would use to arrive at the result.

EXERCISE 12

Orthographic Projection (see Chapter XIV)

Draw the plan and elevation of each of the following geometrical solids of the dimensions given and in the positions stated.

1. A cube of $1\frac{1}{2}$ inches edge resting on the H.P. with a face parallel to the V.P.

2. A cube of $1\frac{1}{2}$ inches edge standing on the H.P. with a face inclined at 30° to the V.P.

3. A cube of $1\frac{1}{2}$ inches edge supported so that a face is inclined at 30° to the H.P. and that face is perpendicular to the V.P.

4. An equilateral triangular prism, sides of the triangular ends $1\frac{1}{2}$ inches and the length of prism $2\frac{1}{4}$ inches standing on a triangular face and having (*a*) a rectangular face perpendicular to the V.P., (*b*) a rectangular face parallel to the V.P.

5. An equilateral triangular prism, sides of the triangular ends $1\frac{1}{2}$ inches and length of prism $2\frac{1}{4}$ inches with a rectangular face on the H.P. and the triangular ends perpendicular to the V.P.

6. An equilateral triangular prism of the same dimension as in Question 5, the prism resting on a rectangular face on the H.P. and the triangular ends inclined at 45° to the V.P.

7. An equilateral triangular prism of the same dimension as in Question 5, the prism standing on a triangular face and having a rectangular face inclined at 30° to the V.P.

8. A hexagonal prism, $1\frac{1}{4}$ inches side of hexagon and $2\frac{3}{4}$ inches long, standing on the H.P. and having a rectangular face (*a*) parallel to the V.P. and (*b*) inclined at 45° to the V.P.

9. A hexagonal pyramid $1\frac{1}{4}$ inches side of hexagon, perpendicular height $2\frac{3}{4}$ inches, standing on the H.P. with (*a*) an edge of base parallel to the V.P. and (*b*) an edge of the base inclined at 45° to the V.P.

10. A hexagonal pyramid, $1\frac{1}{4}$ inches side of hexagon and height

$2\frac{3}{4}$ inches, resting with a triangular face on the H.P. and having an edge of the base at right-angles to the V.P.

11. A hexagonal prism, $1\frac{1}{4}$ inches side of hexagon and length $2\frac{3}{4}$ inches, supported so that an end is inclined at 30° to the H.P. and a rectangular face parallel to the V.P.

12. A hexagonal prism, $1\frac{1}{4}$ inches side of hexagon and length $2\frac{3}{4}$ inches, supported so that an end is inclined at 30° to the H.P. and an edge of the base is at right-angles to the V.P.

13. As in Question 11 but substituting a pentagonal prism for hexagonal prism.

14. As in Question 9 but substituting a pentagonal pyramid for hexagonal pyramid.

EXERCISE 13

Metric Projection (see Chapter XV)

Draw the following solids in (*a*) *Isometric projection,* (*b*) *Axonometric projection,* (*c*) *Oblique projection.*

1. A cube of $1\frac{1}{2}$ inches side.

2. A square prism of $1\frac{1}{2}$ inches side and $2\frac{1}{2}$ inches long.

3. An equilateral triangular prism of $1\frac{1}{2}$ inches side of triangle and $2\frac{1}{2}$ inches long.

4. A square pyramid, $1\frac{1}{2}$ inches side of base and vertical height $2\frac{1}{4}$ inches.

5. A hexagonal prism, $1\frac{1}{4}$ inches side of hexagon and $2\frac{1}{2}$ inches high.

6. A hexagonal pyramid, $1\frac{1}{4}$ inches side of hexagon and $2\frac{1}{2}$ inches high.

7. A pentagonal prism, $1\frac{1}{4}$ inches side of pentagon and $2\frac{1}{2}$ inches long.

8. A pentagonal pyramid, $1\frac{1}{4}$ inches side of pentagon and perpendicular height $2\frac{1}{2}$ inches.

9. A cylinder, 2 inches diameter, $2\frac{1}{2}$ inches long.

10. A cone, 2 inches diameter, $2\frac{1}{2}$ inches perpendicular height.

11. A hemisphere of $1\frac{1}{2}$ inches radius.

EXERCISE 14

POINTS, LINES AND PLANES (see Chapter XVI)

1. Draw the plan and elevation of each of the points *A*, *B*, *C* and *D* when in the positions given:

(*A*) is 1 inch above the H.P. and 1½ inches in front of the V.P.
(*B*) is 1¾ inches above the H.P. and on the V.P.
(*C*) is on the H.P. and 1¼ inches in front of the V.P.
(*D*) is on both the H.P. and V.P.

2. Write down the positions of the points *e*, *f*, *g* and *h* with regard to the H.P. and V.P. (see Fig. 1).

3. Draw the plan and elevation of each of the following lines:

AB, is 2 inches long, parallel to and 1 inch above the H.P. and parallel to and 1½ inches in front of the V.P.

CD is 2½ inches long, inclined at 30° to the H.P. and parallel to the V.P. and having end *C* on the H.P. and 1 inch from the V.P.

EF is 3 inches long on the H.P. end, *E* is ½ inch from the V.P. and the line inclined at 45° to the V.P.

GH is 2 inches long, parallel to and 1 inch above the H.P. and having end *G* on the V.P.

JK, 2½ inches long, perpendicular and ½ inch from the V.P.

4. State the positions of the lines *lm*, *np* and *pq* with regard the H.P. and V.P. (see Fig. 2).

5. Draw the plan and elevation of the lines *AB* and *CD* each 2½ inches long. When their ends are in the positions given end *A* is ½ inch above the H.P. and 1 inch in front of the V.P. End *B* is 1½ inches above the H.P. and the line *AB* is parallel to the V.P. End *C* is ¾ inch above the H.P. and ¼ inch in front of the V.P. The line *CD* is parallel to the H.P. and end *D* is 1¾ inches in front of the V.P.

6. (*a*) What is the position of a line with regard to the V.P. for the elevation to show the true length of a line? What other data does the elevation of the line show?

(*b*) What is the position of a line with regard to the H.P. for the plan to show the true length of the line? What angle does the plan show when the line is in this position?

7. Obtain the true lengths and the true inclinations of each of the lines *pq*, *rs* and *tu*. The plans and elevations of these lines being shown in (Fig. 3).

8. (*a*) What are the traces of a plane?

(*b*) Draw the traces of an inclined plane which is inclined at 45° to the H.P. and is at right-angles to the V.P.

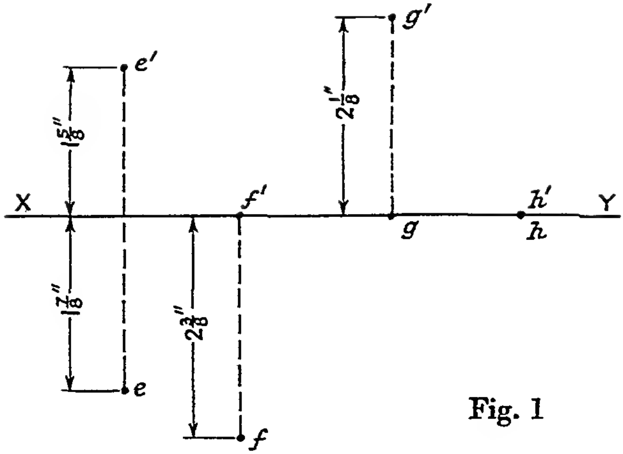


Fig. 1

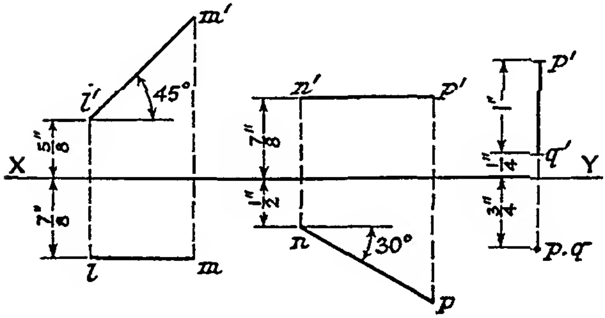


Fig. 2

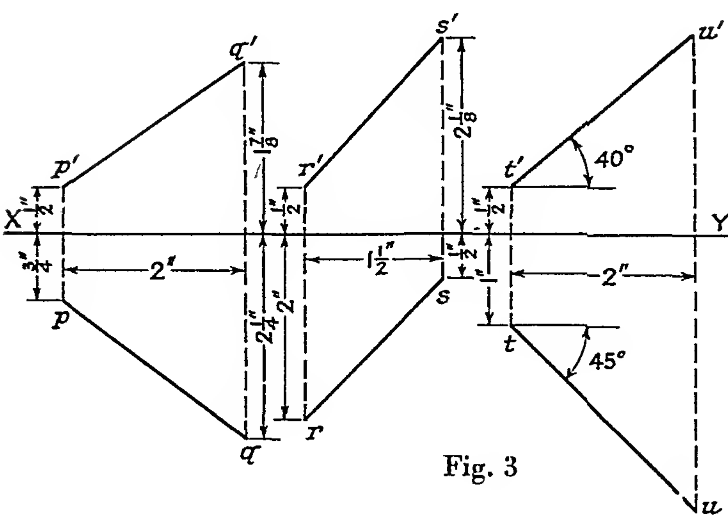


Fig. 3

(*c*) Draw the traces of a vertical plane inclined at 35° to the standard V.P.

(*d*) Draw the traces of a plane which is inclined at 30° to the H.P. and 60° to the H.P.

EXERCISE 15

Inclined Projection of Solids (see Chapter XVII)

1. Fig. 4 shows the plan and elevation of a hexagonal prism resting on the H.P. Obtain the plan and elevation when the diagonal *ab* is inclined from *a* at 30° to the H.P. and the axis parallel to the V.P.

2. Fig. 5 shows the plan and elevation of a right cone resting on the H.P. Obtain the plan and elevation when the base is inclined at 45° to the H.P. and the axis parallel to the V.P.

3. Fig. 6 shows the plan and elevation of a hollow cylinder resting with the base on the H.P. Obtain the plan and elevation when the base is inclined at 30° to the H.P. and the axis parallel to the V.P.

4. Referring to Fig. 4, let the edge *ac* rest upon the H.P. and the axis be inclined at 30° to the V.P. Obtain the plan and elevation of the prism when in this position.

5. Referring to Fig. 5, let the cone rest with one side on the H.P. and its axis parallel to the V.P. Obtain the plan and elevation.

6. Referring to Fig. 5, let the cone rest with one side on the H.P. and its axis inclined to the V.P. at 45°. Obtain the plan and elevation.

EXERCISE 16

Alteration of Ground Line (see Chapter XVIII)

1. (*a*) Draw the plan and elevation of a pentagonal prism, $1\frac{1}{4}$ inches side of pentagon and $2\frac{1}{2}$ inches long, standing on the H.P. with one edge of the base at right-angles to the standard V.P. (*b*) Project a new elevation on a V.P. which is inclined at 30° to the standard V.P.

2. (*a*) Draw the plan and elevation of a tetrahedron the base of which is an equilateral triangle of 2 inches side and vertical height $2\frac{1}{2}$ inches, when standing on the H.P. with one edge of the base parallel to the standard V.P. (*b*) Draw a new elevation on a V.P. inclined at 45° to the standard V.P. and (*c*) draw an end elevation.

3. (*a*) Draw th plan and elevation of a regular hexagonal prism, $1\frac{1}{4}$ inches side of hexagon and $2\frac{1}{2}$ inches long, standing on the H.P.

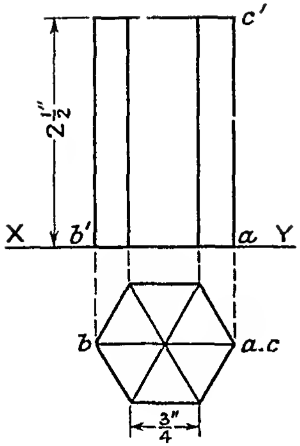


Fig. 4

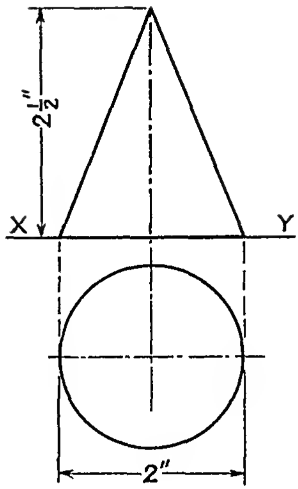


Fig. 5

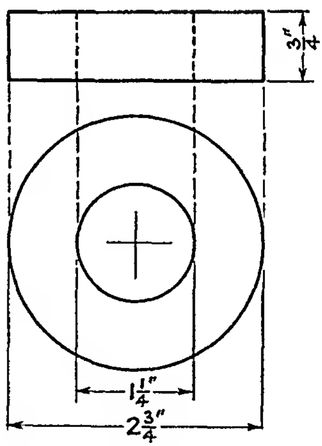


Fig. 6

with one rectangular face parallel to the V.P. (*b*) Draw a new plan on the XY which is inclined at 45° to the original XY line.

4. As in Question 3 but substituting a hexagonal pyramid for the hexagonal prism.

5. Draw as in part (*a*) of Question 2, (*b*) Project a new plan on a new XY inclined at 30° to the original XY line.

6. Draw as in part (*a*) of Question 1, (*b*) project a new plan on a new XY inclined at 45° to the original XY line.

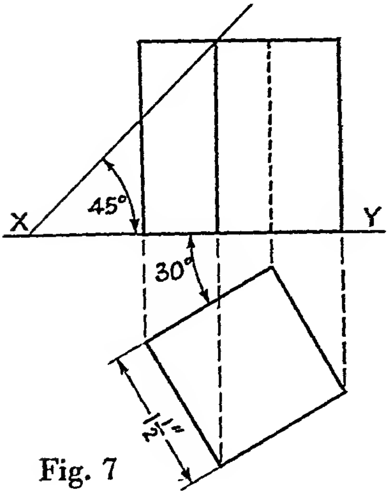


Fig. 7

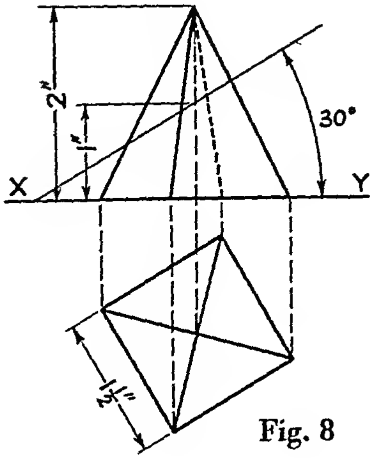


Fig. 8

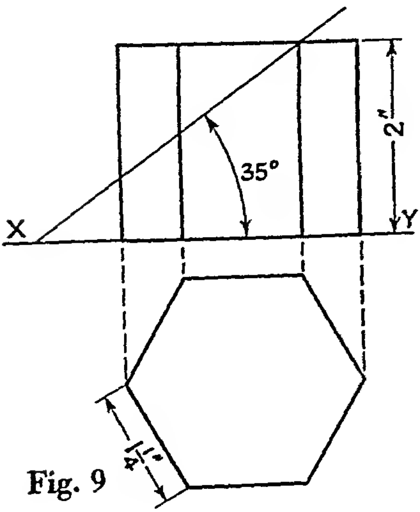


Fig. 9

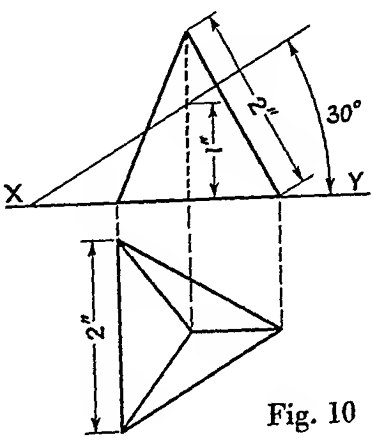


Fig. 10

EXERCISE 17

Sections of Solids and Development of Surfaces
(see *Chapter XIX*)

1. Figs. 7, 8, 9, 10, 11 and 12 show the plans and elevations of various geometrical solids cut by inclined planes. Draw, in each case, the plans and elevations to the dimensions given and obtain the plan and true shape of the section made by the inclined plans.

2. Draw the plan and elevation of a right cone diameter of base $2\frac{1}{2}$ inches and vertical height $3\frac{1}{2}$ inches and obtain the plan and true shape of the section when cut by (*a*) a plane parallel to and $\frac{1}{2}$ inch from an outside and inclined edge of the cone, (*b*) a vertical plane at right angles to the standard V.P. and $\frac{3}{4}$ inch from the axis of the cone.

3. Draw the true shape and plan of the section of a sphere, of $2\frac{3}{4}$ inches diameter, made by a plane inclined at 45° to the H.P. and at right-angles to the standard V.P. The plane cuts the sphere at a distance of $\frac{3}{4}$ inch from the centre.

4. Draw the development (*a*) of the whole of the inclined faces of the square pyramid shown in Fig. 8. (*b*) The surface of the pyramid below the cutting plane.

5. Draw the development (*a*) of the surface of the tetrahedron shown in Fig. 10. (*b*) The surface of the tetrahedron below the cutting plane.

6. Draw the development of the curved surface of that part of the cylinder shown in Fig. 11.

7. Draw the development of (*a*) the curved surface of the cone shown in Fig. 12. (*b*) *The curved surface below the cutting plane.*

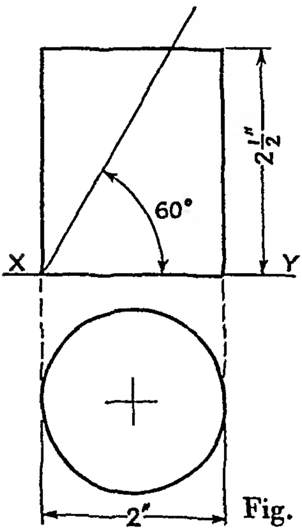


Fig. 11

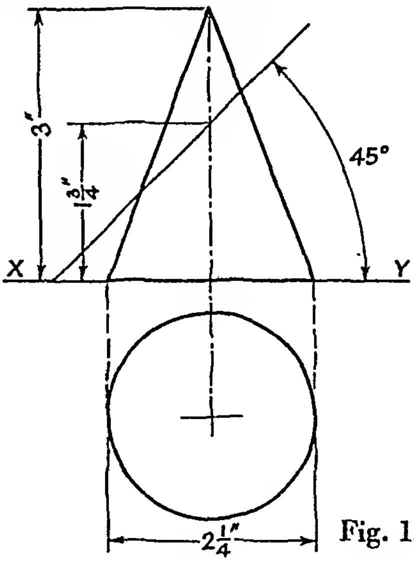


Fig. 12